

# **CIT 5920 — Lecture 23: Proof Medley & Intro to Graphs**

03 - 03 Dec 2024

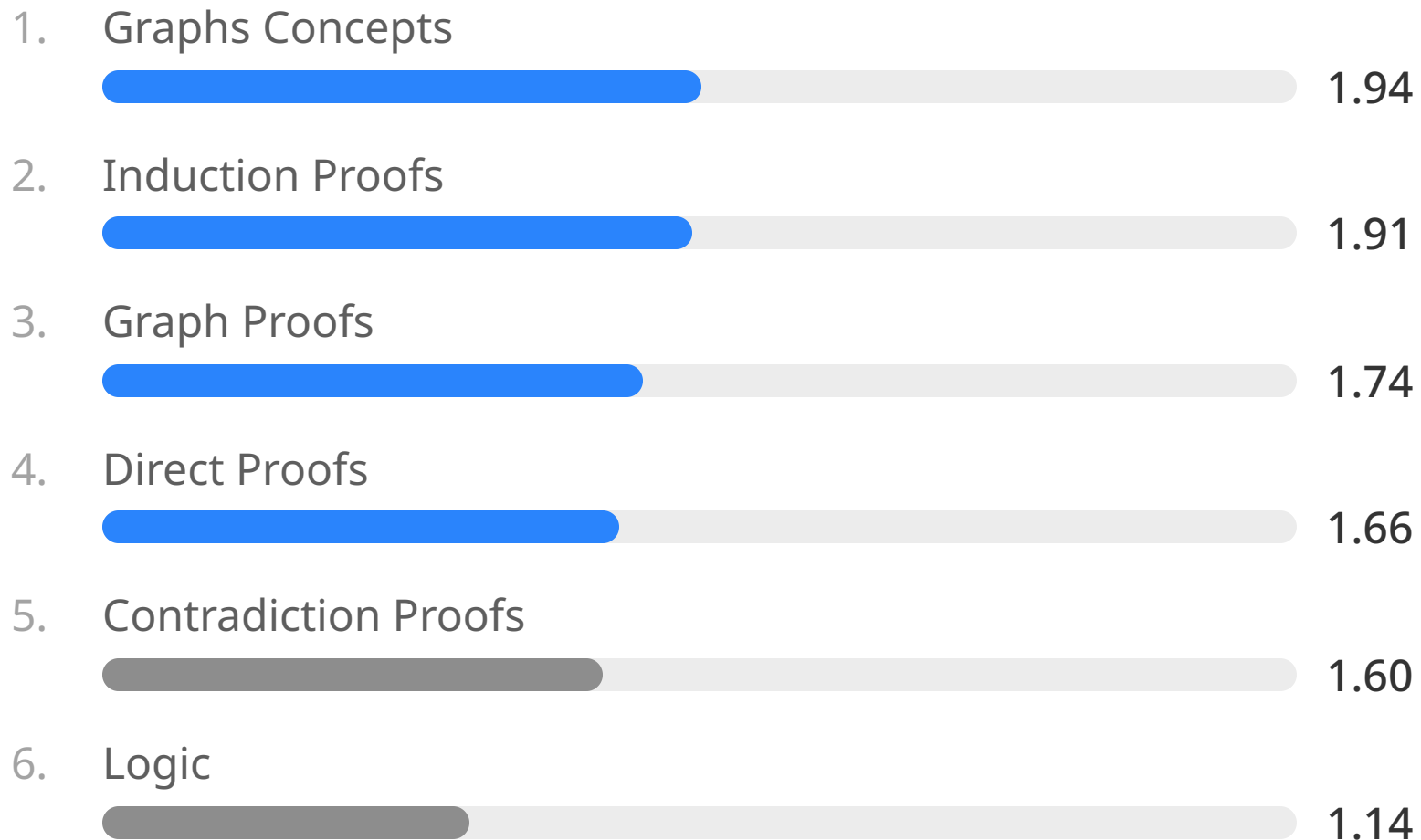
Poll results

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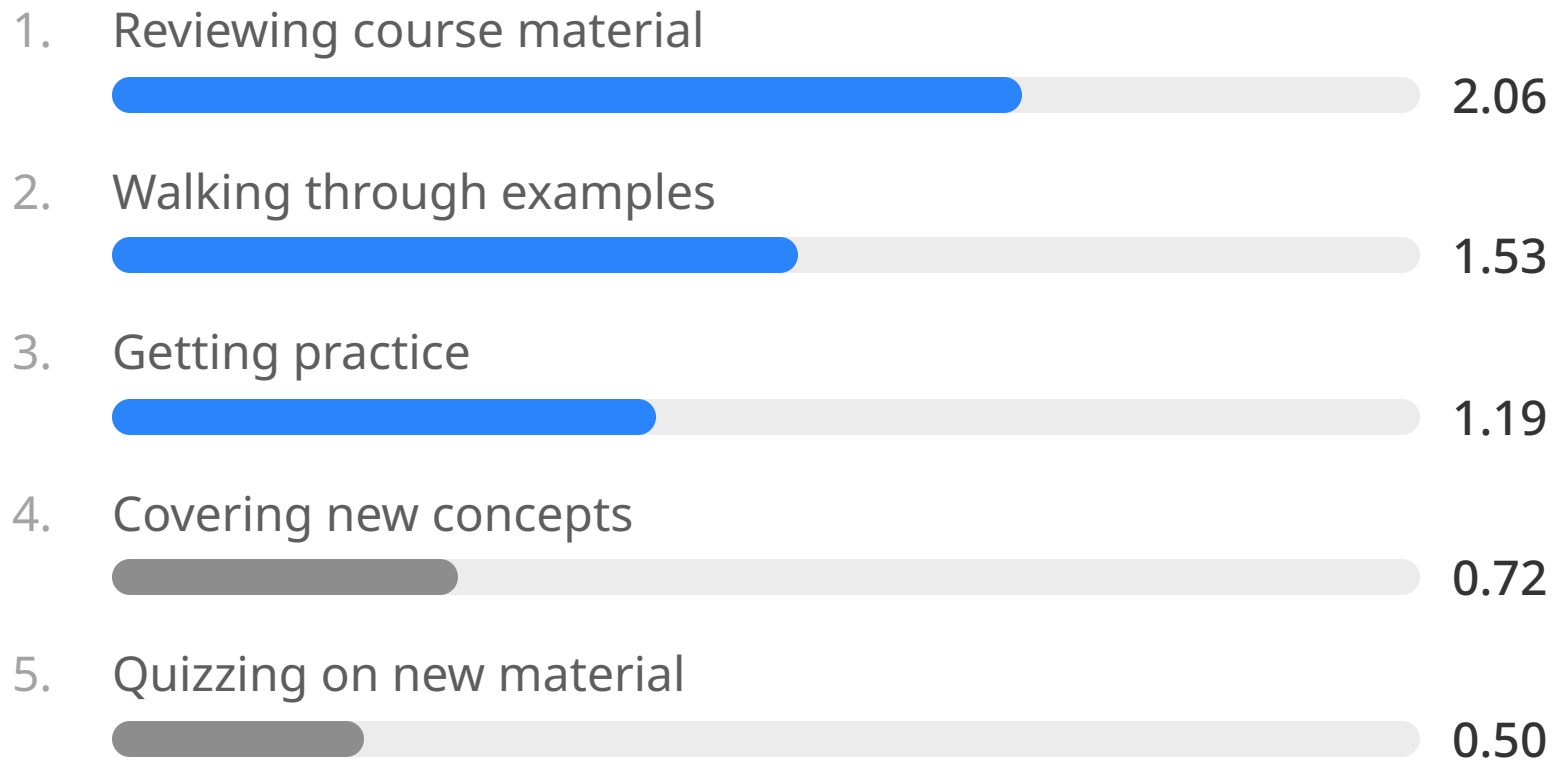
## Which topics would you like to **\*DISCUSS\*** today?

035



## What are you in the mood for?

036



## What is the topic that you are most uncertain about?

(1/3)

029

- all proofs
- Induction
- Why universal quantifier takes  $\rightarrow$  and existential quantifier takes  $\vee$  when it comes to modifying Eng into logic.
- I need clarification on concepts about proofs and more examples and practices.
- proof by contrapositive
- I would like you to help re-cover again mainly proofs. Everything from direct, contradiction, contrapositive, and induction. Doesn't have to go super slow but help get an understanding of the right mindset in tackling these proofs, the format (if possible) of each proof to make things easier to tackle new problems.
- I am confused about almost all the topics after the midterm, please HELP. 🤯🤔😞
- For one, set proofs because I am not sure when we can rely on previously covered logic laws or when we have to

## What is the topic that you are most uncertain about?

0 2 9

(2/3)

- elaborate. Also, since graphs are a new concept, I want to learn about graph proofs if they are to be tested
- What sort of things we can "assume" or use during proofs (sometimes it feels like the things are random, aren't sure we can use) Also, probability, and to what extent will they be covered on the exam
- Hard to choose one, generally most uncertain about the final exam since I feel unprepared on so many topics.
- Logic, proofs
- Want to review the graph theory, and do some practice on the induction proof
- Strong induction
- Induction and also, not sure I understand what the best way to write a proof is
- How to convert the natural language format into a math format and convert it into a reverse logic.
- Rigorous math definition of each topic in those proofs
- I would like more examples

## What is the topic that you are most uncertain about?

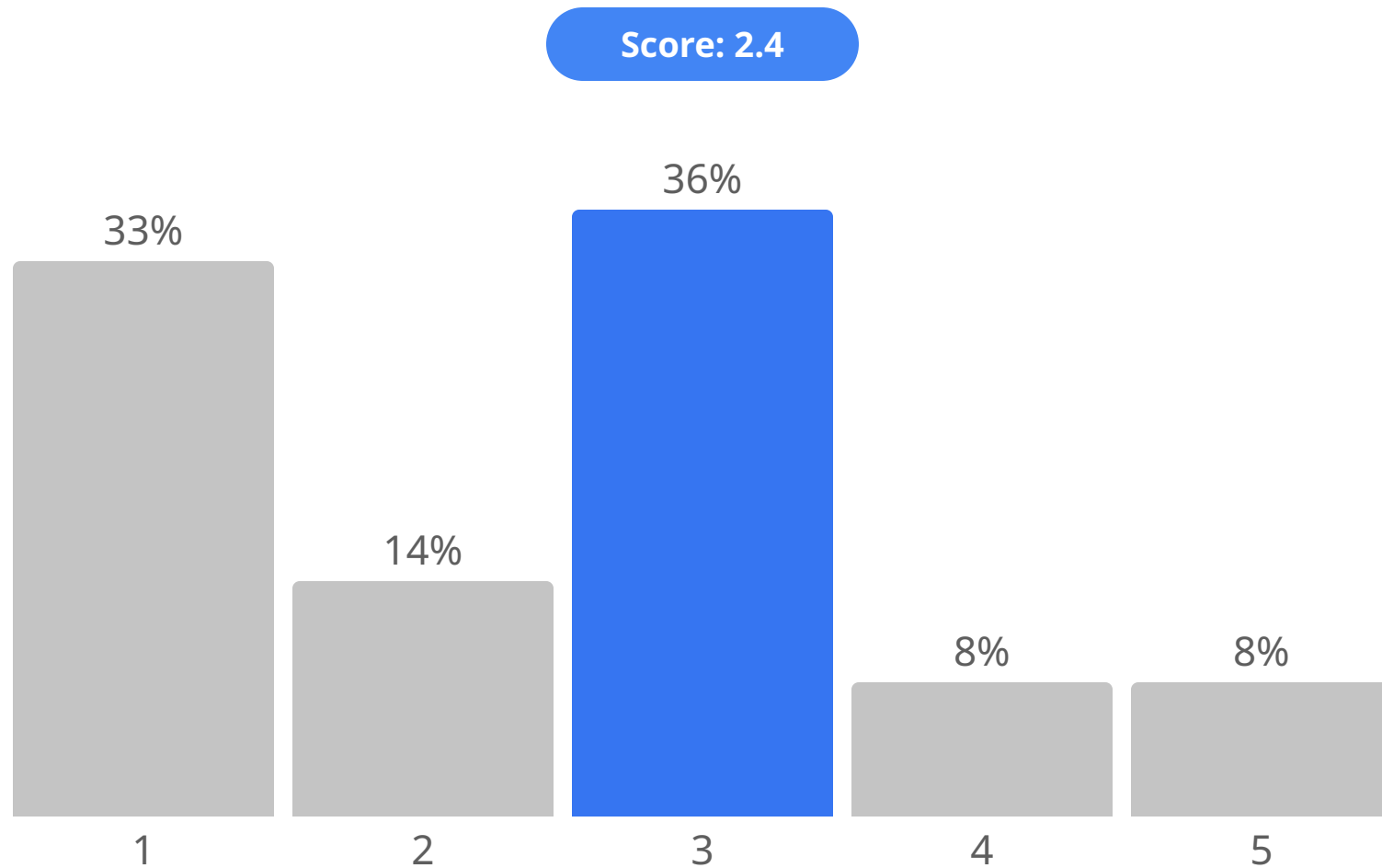
(3/3)

0 2 9

- of proofs like the ones in HW6 and how to approach proof questions. It is still unclear to me and having some clear examples would be very useful
- graph theory
- Proof
- Proofs
- logic
- Proofs Independent Variables in Probability
- Have a lot of topics uncertain
- Logic and proofs
- Graph
- I feel I am lost.
- Proofs
- graph
- Graph theory

Overall what is your confidence level in terms of discrete math?

036





## What is a graph composed of?

Vertices and edges ✓



Numbers and letters

3 %

Circles and squares

3 %

Lines and bars

9 %

**In a graph  $G = (V, E)$ , what does  $V$  represent?**

Edges

☐ 0 %

Vertices ☒

☒ 97 %

Paths

☐ 3 %

Quiz: Graph Basics (3/10)

037

**In the notation  $G = (V, E)$ , what does  $E$  represent?**

Edges ✓



Vertices

☐ 0 %

Paths

☐ 0 %

## What is a complete graph?

A graph with only one vertex

☐ 3 %

A graph where every vertex is connected to every other vertex ✓

☒ 97 %

A graph with no edges

☐ 0 %

**The "vertices" refers to the:**

the round dots ✓



the lines between the dots



something else



**The "edges" refers to the:**

the round dots

☐ 0 %

the lines between the dots ☒

☒ 100 %

something else

☐ 0 %

## What is a loop in graph theory?

A path that starts and ends at the same vertex



An edge that joins a vertex to itself ✓



A graph with circular edges



Quiz: Graph Basics (8/10)

031

**In a graph, if vertex A is connected to vertex B, what is this connection called?**

A path



An edge ✓



A degree





Quiz: Graph Basics (9/10)

037

**How many edges are in a complete graph with 5 vertices?**

10 ✓



84 %

15



8 %

20



8 %

## What is a directed graph?

A graph where each edge implies a direction from one vertex to another



A graph with no loops



A graph where all vertices are connected



Quiz: Graph Medley (1/15)

0 3 4

**What is the largest degree of a vertex in a graph with vertices  $\{X, Y, Z\}$  and edges  $\{\{X, Y\}, \{Y, Z\}\}$ ?**

1

☐ 3 %

2 ☒

☐ 85 %

3

☐ 6 %

4

☐ 6 %

Quiz: Graph Medley (2/15)

033

**How many edges are in a complete graph with 6 vertices?**

5

☐ 3 %

10

☐ 3 %

15 ☒

☐ 82 %

26

☐ 6 %

30

☐ 6 %

Quiz: Graph Medley (3/15)

0 3 5

**In a graph with vertices  $\{A, B, C, D\}$  and edges  $\{\{A, B\}, \{B, C\}, \{C, D\}\}$ , what is the length of the longest path?**

2

☐ 3 %

3 ☒

☒ 86 %

4

☐ 11 %

5

☐ 0 %

## What is a self-loop in a graph?

An edge connecting a vertex to itself ✓



An edge connecting two distinct vertices



A path that starts and ends at the same vertex



A cycle in the graph



Quiz: Graph Medley (5/15)

036

**Which of the following is a property of an undirected graph?**

If  $(u, v)$  is an edge, then  $(v, u)$  is not an edge

☐ 14 %

If  $(u, v)$  is an edge, then  $(v, u)$  is also an edge ✓

☒ 83 %

Edges have a direction

☐ 0 %

Edges cannot connect a vertex to itself

☐ 3 %

Quiz: Graph Medley (6/15)

036

**In a graph with vertices  $\{P, Q, R, S\}$  and edges  $\{\{P, Q\}, \{Q, R\}, \{R, S\}, \{S, P\}\}$ , is there a cycle?**

Yes ☒



No

☐ 0 %



Quiz: Graph Medley (7/15)

0 2 8

**What is the degree sequence of a graph with vertices  $\{1, 2, 3, 4\}$  and edges  $\{(1, 2), (2, 3), (3, 4), (4, 1), (1, 3)\}$ ?**

(4, 3, 2, 1)

 11 %

(3, 2, 2, 1)

 29 %

(2, 2, 2, 2)

 0 %

(3, 3, 2, 2) ✓

 61 %

Quiz: Graph Medley (8/15)

031

**In a graph with vertices  $\{A, B, C, D, E\}$  and edges  $\{\{A, B\}, \{B, C\}, \{C, D\}, \{D, E\}\}$ , what is the minimum number of edges to add to make it a complete graph?**

1

 6 %

4

 10 %

6 ☒

 68 %

10

 16 %

Quiz: Graph Medley (9/15)

028

**Can two graphs be isomorphic if they have different numbers of vertices?**

Yes

 7 %

No ☒

 57 %

Did not cover graph isomorphism

 36 %

Quiz: Graph Medley (10/15)

0 2 8

**What is the degree sequence of the graph with vertices  $\{M, N, O, P\}$  and edges  $\{(M, N), (N, O), (O, P), (P, M)\}$ ?**

1, 1, 1, 1

☐ 4 %

2, 2, 2, 2 ☒

☒ 96 %

3, 3, 1, 1

☐ 0 %

4, 4, 2, 2

☐ 0 %

Quiz: Graph Medley (11/15)

030

**What is the sum of the degrees of all vertices in a graph with 4 vertices and 3 edges?**

3

 7 %

4

 13 %

6 ☒

 63 %

8

 0 %

Not enough information

 17 %

## What is a planar graph?

A graph that has the shape of an airplane

 10 %

A graph that contains cycles

 3 %

A graph that contains crossing edges

 6 %

A graph that can be drawn without crossing edges ✓

 81 %

Quiz: Graph Medley (13/15)

0 3 4

**If a graph has vertices  $\{A, B, C, D, E\}$  and edges  $\{(A, B), (B, C), (C, D), (D, E), (E, A)\}$ , is it planar?**

Yes ✓



No



Quiz: Graph Medley (14/15)

030

**In a graph with vertices  $\{A, B, C, D\}$  and edges  $\{(A, B), (B, C), (C, D)\}$ , which vertices are endpoints of the edge  $(B, C)$ ?**

A and B

☐ 0 %

B and C ☒

☒ 90 %

C and D

☐ 0 %

A and D

☐ 10 %



Quiz: Graph Medley (15/15)

0 3 1

**How many paths exist between vertices A and D in a graph with vertices  $\{A, B, C, D\}$  and edges  $\{(A, B), (B, C), (C, D), (A, D)\}$ ?**

1



2 ✓



3



4

