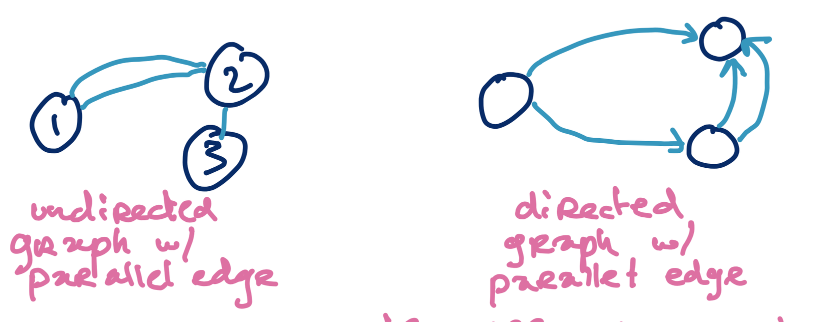
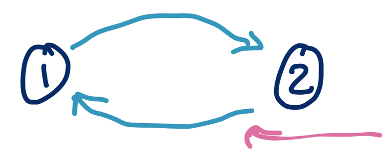
Lecture 24

Quick answers

- parallel edges can be directed or undirected



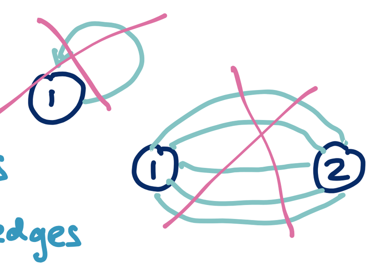
- not parallel edge if different direction



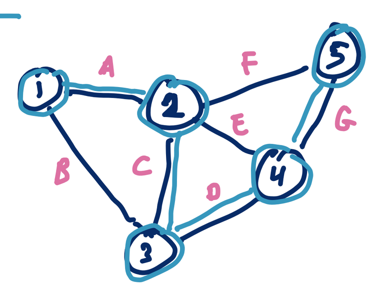
not a parallel edge
$(1,2) \neq (2,1)$
$(A,B) \neq (B,A)$

In this course, unless explicitly mentioned, we are interested in graphs that are:

- undirected
- no self-loops
- no parallel edges
- finite number of edges (and vertices)



PROPERTIES OF GRAPHS



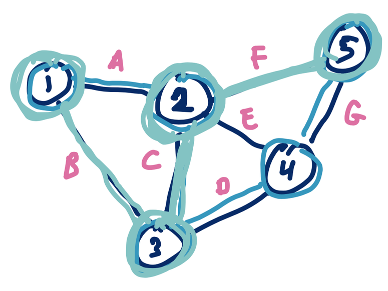
Def.: A path is an alternating sequence of vertices and edges, starting and ending in a vertex, and no vertex appears more than once.*

this is an alternating sequence

$\boxed{1\ A\ 2\ C\ 3\ D\ 4\ G\ 5}$
(example of a path from 1 to 5)

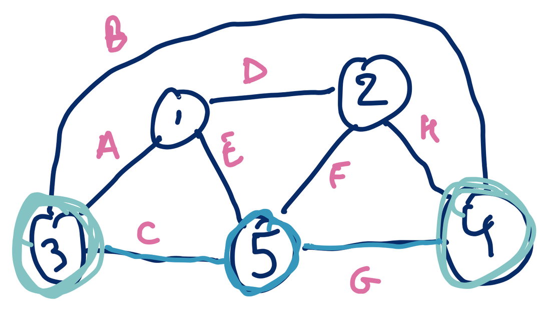
(* therefore a path cannot contain a loop)

Def.: A walk is
- an alternating seq. of vertices and edges,
- starting and ending in a vertex.



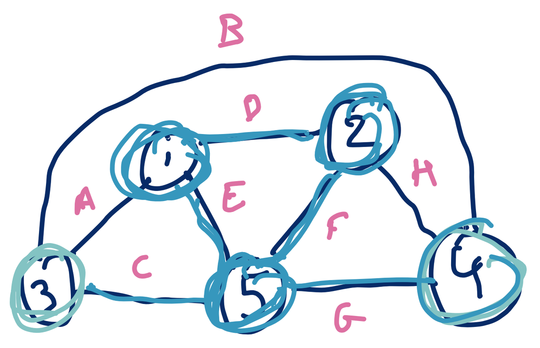
1 B 3 C 2 F 5
(another path 1 to 5)

Question: What is the difference between a path and a walk?



3 C 5 G 4

PATH = no repetition



$(v_1, e_1, v_2, e_2, v_3, e_3 \ldots)$

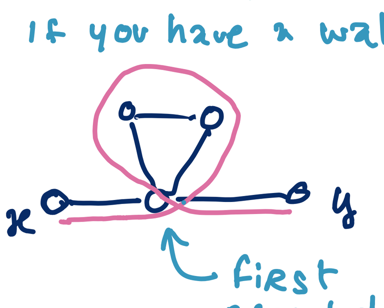
3 C 5 E 1 D 2 F 5 E 1 D 2 F 5 E 1 D 2 F
5 E 1 D 2 F 5 E 1 D 2 F 5 E 1 D 2 F 5 G 4

WALK = allows for repetition

Corollary: Every path is a walk.

Thm: If there is a WALK between two vertices x and y, there must be a PATH between x and y.

If you have a walk:

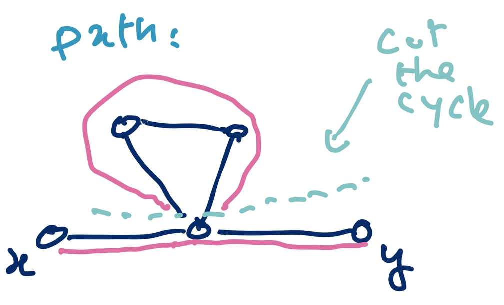


first repeated vertex

Def.: A CYCLE is a walk that starts and ends in the same VERTEX.

Begin by considering any walk between vertices x and y. This walk can be categorized in two cases:

you can simply "cut" the cycle and you get a path:



CASE 1 The walk does not contain repeated vertices.
- in this case, the walk already qualifies as a path, by definition.

CASE 2 The walk includes at least one vertex that is repeated.
- Let's call z the first repeated vertex.

$w = (\ldots, z, e_1, \ldots, e_k, z, \ldots)$

formal notation to represent PATH / WALK / CYCLE

e_i this is an edge

$w = (v_1, e_1, v_2, e_2, v_3, e_3 \ldots)$

this is vertex v_i

- Create a shorter walk by eliminating the segment of the walk between the first and last occurrences of z

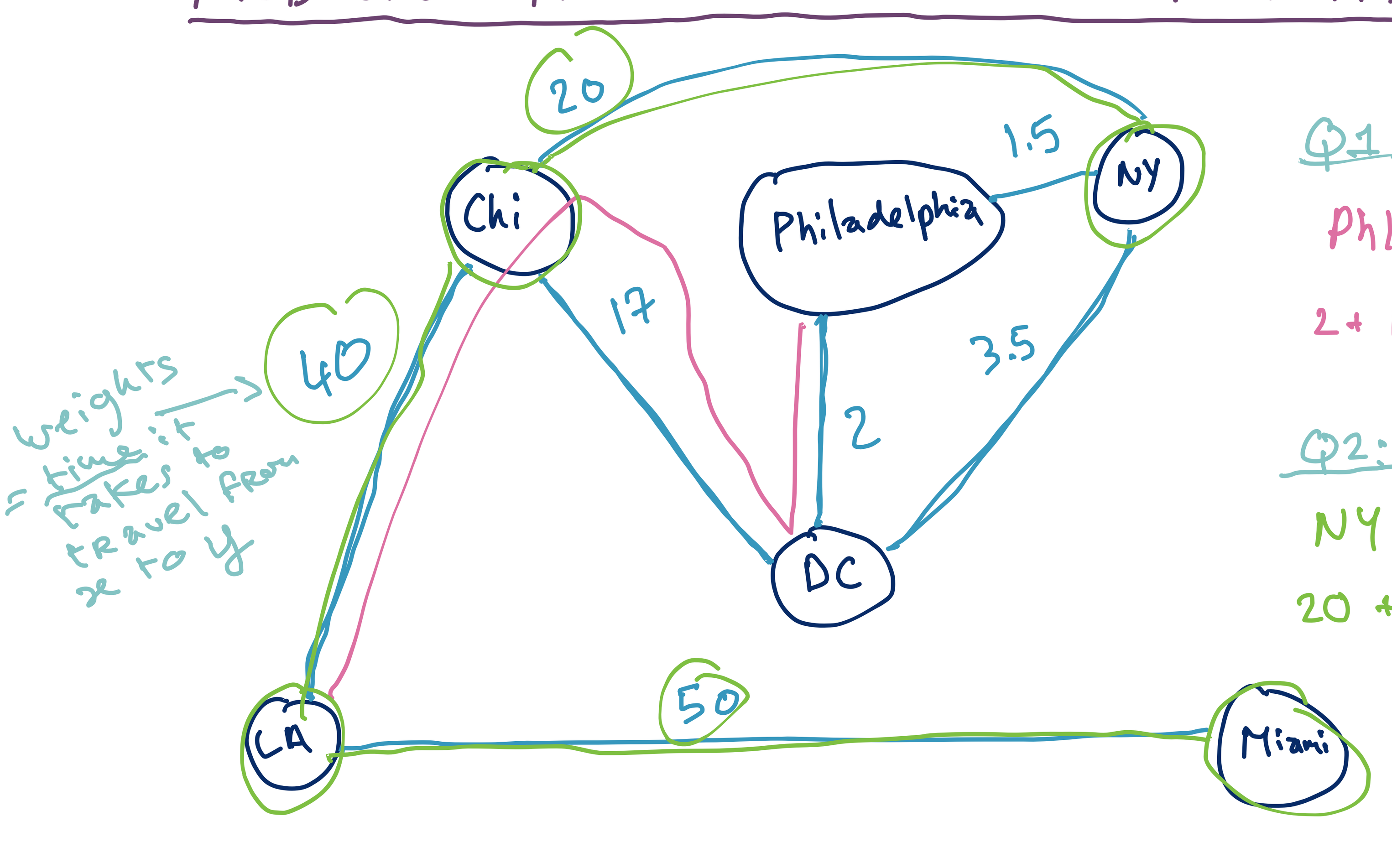
$w' = (\ldots z \ldots)$

Specifically, we only retain the first occurrence of z. The resulting is strictly shorter, because we have removed at least one vertex [Argument to avoid infinity.]

- Continue this process iteratively until there are no more repeated vertices.
- Since each iteration removes at least one vertex, and there are a finite number of vertices, this process will eventually result in a walk with no repeated vertices — that is, a path!

CONCLUSION:
- The final walk goes from x to y without repeating vertices, thus meeting the def. of a path
- This process proves that if there is a walk between two vertices, there must also be a PATH. ∎

MODELING THE RAIL NETWORK WITH GRAPHS



Q1: Phl → LA ?

Phl → DC → Chi → LA

$2 + 17 + 40 = \underline{59}$

Q2: NY → Miami

NY → Chi → LA → Miami

$20 + 40 + 50 = \underline{110}$