CIT5920 Recap Lecture

SETS

1. Set	(unordered) collection [of distinct objects]	$A = \{1, 2, 3\}$ $= \{3, 2, 1\}$ $= \{3,3,3,2,1,1\}$
2. Element	member of collection	
3. Subset	A is a subset of B, if all elements in A are also in B	
4. Union.	the union of A and B is element in A OR B	
5. Intersection.	— intersection — A AND B	
6. Complement	what is not in the set	
7. Cardinality	the number of elements in a set	
8. Power Set	all the ways of forming subsets of set	

INDUCTION
- define P(n)
- base
- induction

Question Prove by induction that the number of subsets of a set with n elements is 2^n, $n \geq 0$, $n \in \mathbb{Z}$

- **PREDICATE**: $P(n)$: a set with n elements has 2^n subsets
- **BASE CASE**: for $n=1$, $S = \{a\}$ for instance, S has one element and we can form 2 subsets: $\varnothing$ and $\{a\}$
- **INDUCTIVE CASE**: Suppose $P(k)$ is true, we will show this implies $P(k+1)$.
  Let S be a set with $k+1$ elements. Let x be some element of S, and consider $S' = S \setminus \{x\}$. S' has exactly k elements, therefore we can apply the inductive hypothesis, and S' has 2^k subsets.
  From the 2^k subsets of S', we can form $\boxed{2} \cdot 2^k$ subsets of S (the original set), because for each subset of S', we can either add x or not (two choices)
  Since $P(1)$ is true and $P(k) \Rightarrow P(k+1)$, this prop.'s true for all $n \geq 1$.

CORE OF PROOF in which we reduce S so we can apply the inductive hypothesis

RELATIONS

1. Relation [a collection (=set) of ordered pairs] = subset of $A \times B$ (Cartesian product)
   1R2 3R4
   $R = \{(1,2), (3,4)\}$
2. Reflexive: R is reflexive (on a set S) if for all $x \in S$, xRx
3. Symmetric: if aRb then bRa
4. Transitive: if aRb and bRc then aRc
5. Equivalence relation: reflexive, symmetric, transitive =
6. Anti-symmetric:
   - if aRb and $a \neq b$ then we don't have ~~bRa~~
   - if aRb and bRa then $a = b$

Example
Let $S = \{1,2,3\}$
- $R_R = \{(1,1), (2,2), (3,3)\}$ is reflexive.
- $R_S = \{(1,2), (2,1)\}$ is symmetric.

Question Consider the relation R on the set $A = \{1,2,3,4,5\}$ defined by $R = \{(1,2), (2,3), (3,4), (4,5), (1,5)\}$.

1. Determine whether R is: reflexive? symmetric? transitive? antisymmetric?
2. Modify R minimally to make it an equivalence relation.

- REFLEXIVE? no because it does not contain the diagonal pairs (1,1), (2,2), (3,3) .... (5,5)
- SYMMETRIC? no, it is missing symmetric pairs like (2,1)
- TRANSITIVE? no, it has (2,3) and (3,4), but is missing (2,4)

ANTI-SYMMETRIC? yes because we never have (a,b) and (b,a) at the same time in R

$R = \{(1,2), (2,3), (3,4), (4,5), (1,5), (1,1), (2,2), (3,3), (4,4), (5,5), (2,1), (3,2), (4,3), (5,4), (5,1), (1,3), (2,4), (3,5), (4,1), (1,4), (3,1), (4,2), (5,3), (2,5)\}$

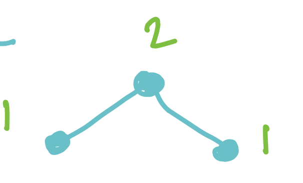
✓ Reflexive
✓ symmetric
✓ transitive
equivalence Relation

GRAPHS

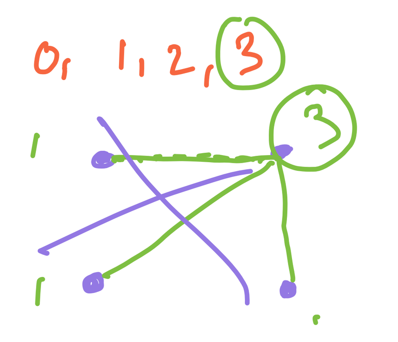
1. Graph (NO SELF LOOPS OR PARALLEL EDGES)
2. Vertex (node)
3. Edge
4. Path — alternating sequence of vertex and edge with no repetition
5. Cycle — a path that starts and ends in the same vertex
6. Connected graph
7. Complete graph
8. Bipartite graph.

Question
Which of these degree sequences are valid degree sequences? If they are not provide a reason, otherwise draw a graph.

- 1, 1, 2



- 0, 1, 2, 3



cannot have a vertex of degree 3 AND of vertex of degree 0 at same time

- 1, 2, 3, 3

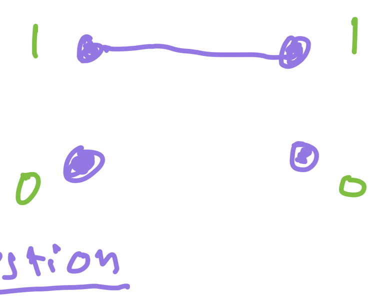
INVALID [Because of the handshake lemma ($\sum \deg(v) = 2|E|$), the sum of degrees is always even
$1+2+3+3 = 9$ is odd

- 3, 3, 1, 1

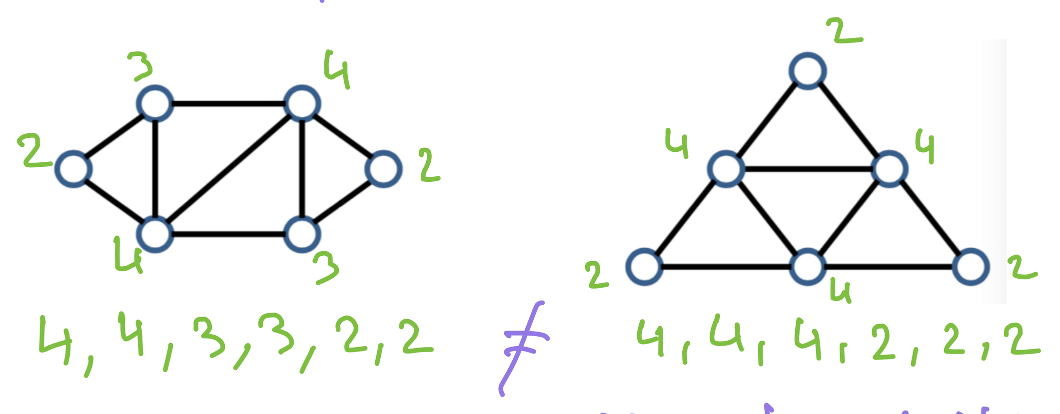
$\{3,1,3,1\}$
1 3 3 1

INVALID for same reason as above

- 0, 0, 1, 1



Question
Are these two graphs isomorphic?



4, 4, 3, 3, 2, 2 $\neq$ 4, 4, 4, 2, 2, 2

the degree sequences are different and therefore the graphs are not isomorphic.

PROBABILITY (discrete)

1. Experiment
2. Sample Space (S)
3. Event
4. Probability of an Event
5. Conditional probability
6. Random Variable
7. Expected Value
8. Linearity of Expectation

Question In a standard deck of 52 cards (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King — for four suits), you draw a single card. What is the probability of drawing a KING or a HEART ("heart" is one of suits).

Let D be the full deck. Let K be the set of KINGS (there are 4 kings) and let H be the set of HEARTS (there are 13 hearts). What we want to compute is:

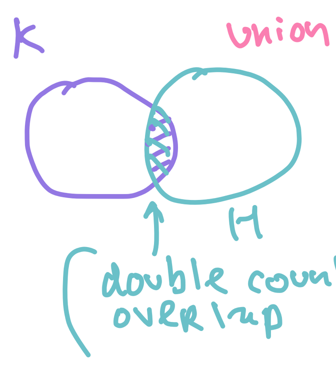
$$\mathbb{P}[\text{KING or HEART}] = \frac{|K \cup H|}{|D|}$$

← the events we are interested in
← all events

$$|K \cup H| = |K| + |H| - |K \cap H|$$

union
the set of kings that are also hearts

$$= 4 + 13 - 1$$
$$= 16$$



(double counting overlap)

$$\mathbb{P}[\text{KING or HEART}] = \frac{16}{52}$$