Graphs

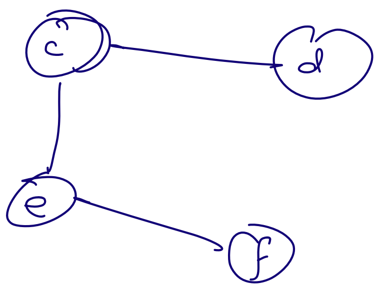
- vertices
- edges
- directed / undirected
- simple graphs
- weighted graphs
- self loops
- types of graphs - cycle, complete, path
- Handshaking lemma
- graphviz

Graph - structure to represent relationships

$G = (V, E)$

V - vertices

E - edges



$E = \{(c, e), (e, f), (c, d), (a, b)\}$

Example

vertex represents person

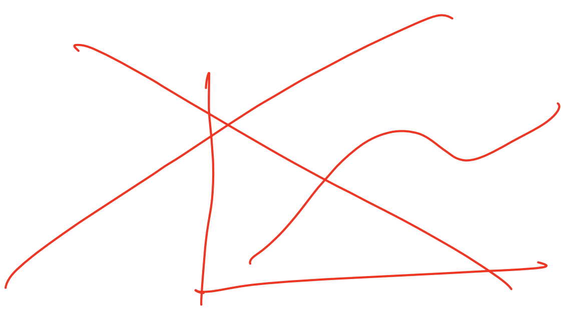
edge $\Rightarrow$ these ppl have eaten lunch together

V - vertex set - $\{v_1, v_2, \ldots, v_n\}$

E - edge set - $E \subseteq V \times V$

Cartesian product

weighted graph v/s unweighted



Relations

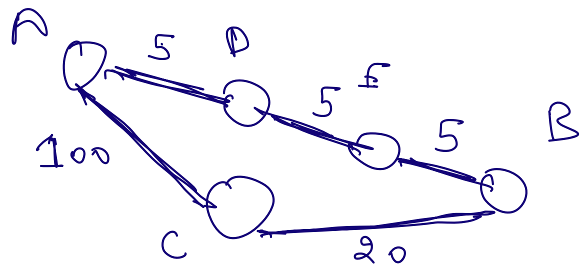
Functions

applications

① Routing problems

network of computers

route info (packet) from comp A to comp B



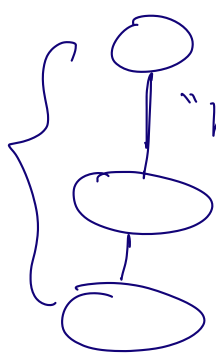
cities and roads

shortest path from city A to B.

② Social network

degrees of separation

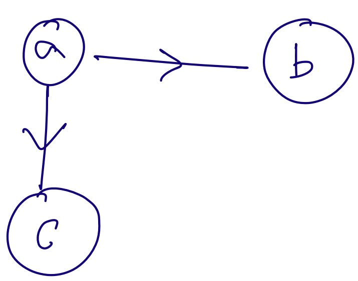
2 degrees of separation



weighted graph

each edge has a weight which is notion of traffic/extent of a relationship.

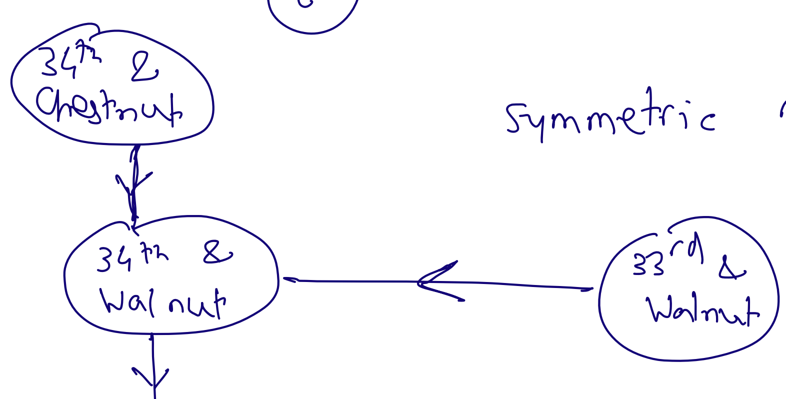
directed and undirected



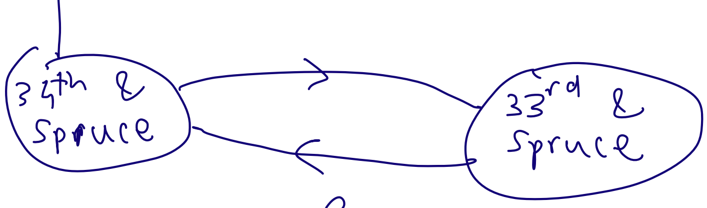
from a to b

$E = \{ (a, b), (a, c) \}$

from   to



symmetric relations ⟷ undirected



OR

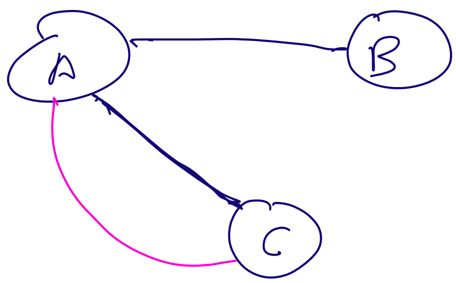
by definition if G contains any directed edge it is directed graph.

Simple graph

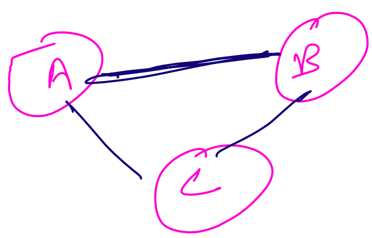
$G = (V, E)$ assume V is finite set

at most 1 edge between any pair of vertices

<u>example</u> islands vertices
bridges edges



not simple graph.



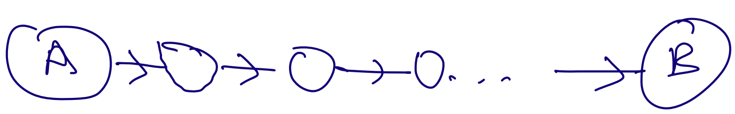
✓ simple

Connected graph v/s disconnected graph

<u>Path</u> — path from vertex A to vertex B is a sequence of vertices

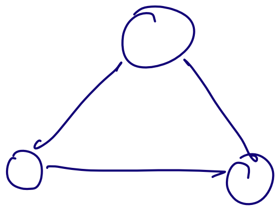
$A, v_1, v_2, \ldots, v_k, B$

such that $(A, v_1), (v_1, v_2), (v_2, v_3) \ldots$ edges

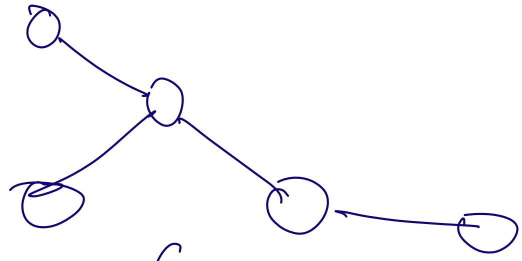


Connected graph

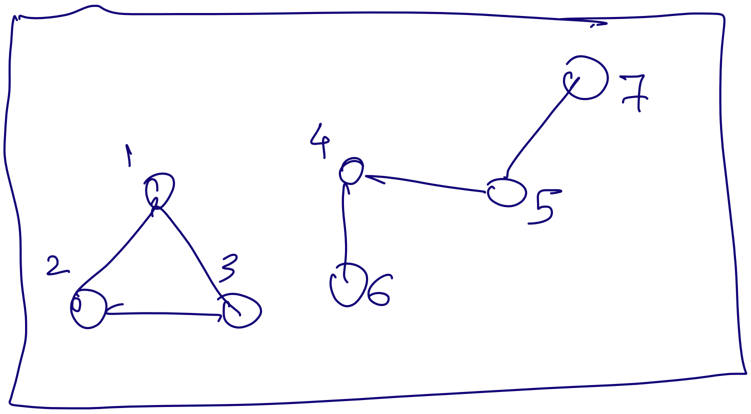
G is connected iff
1. G is undirected
2. $\forall\, u, v \in V\ \exists$ path from u to v.



G_1

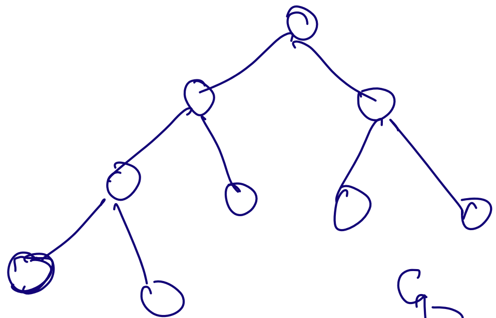


G_2



G_4

not connected
bcoz
consider 3 & 6



G_3

note: Node / vertex
synonyms.

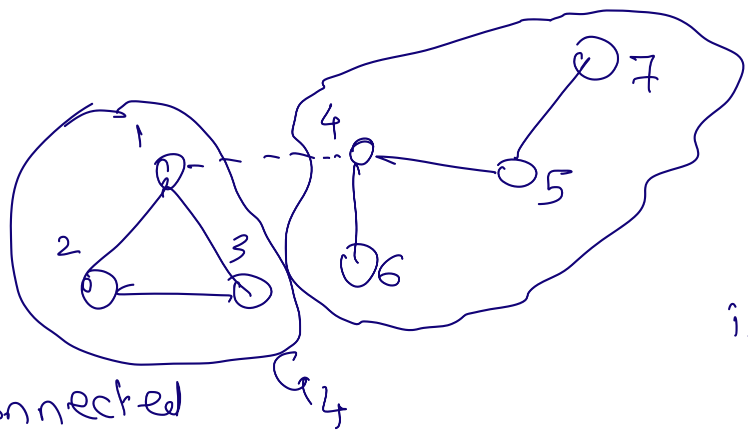
Connected components

if graph is connected then 1 connected component

disconnected graph can be split into connected component
↓
subgraph that is connected &
is maximal in terms of connectivity

subgraph → portion of original graph.

addition of any more vertices makes you lose connectivity



2 connected components

if (u,v) is an edge we say u is <u>adjacent</u> to v

adjacent means edge btwn vertices
connected " path btwn vertices.

Q. G has 3 connected components (c.c.)
now we add an edge to G
(u,v) u & v are in different connected components at the time of adding edge

How many c.c. after (u,v) addition.

<u>Result</u> if we had n c.c. and we add edge (u,v) where u & v are in diff c.c. then # of c.c. is $n-1$

<u>Proof</u> Consider a and b which were in diff connected components. a is in

the same c.c. as u. b is in the same c.c. as v.

Now when (u,v) edge is added we will show a & b become connected i.e. path from a to b.

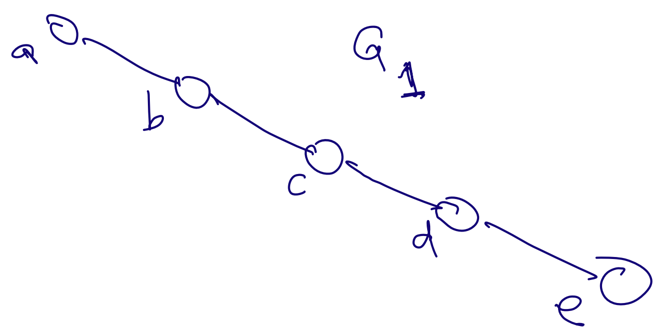
Since u is in a's c.c. ∃ path from a to u $a \rightsquigarrow u$

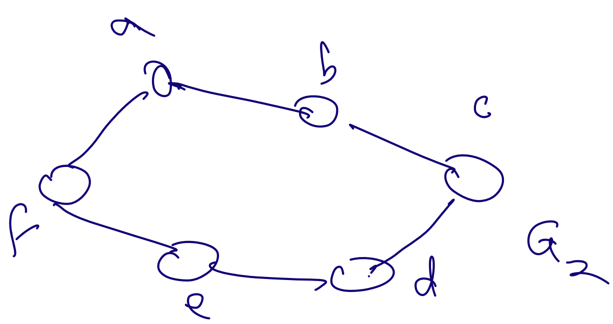
then take edge (u,v)

since v is in b's c.c. ∃ path from v to b.

$$\underbrace{a \rightsquigarrow u \rightarrow v \rightsquigarrow b}_{\text{path from } a \text{ to } b}$$

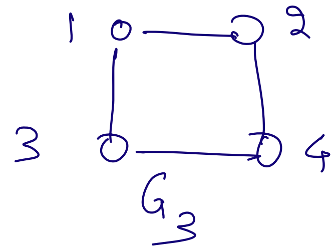
Q. G is a connected graph and we remove an edge does G stay connected?

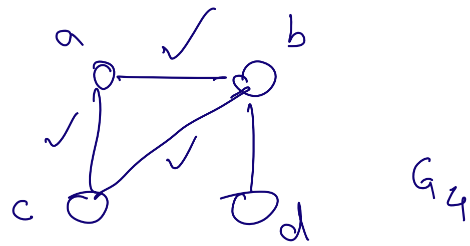




If deletion/removal of an edge still maintains G's connectivity then ______ is in G.

Cycle in a graph is a sequence of vertices & edges between those vertices such that you start and end at the same vertex.





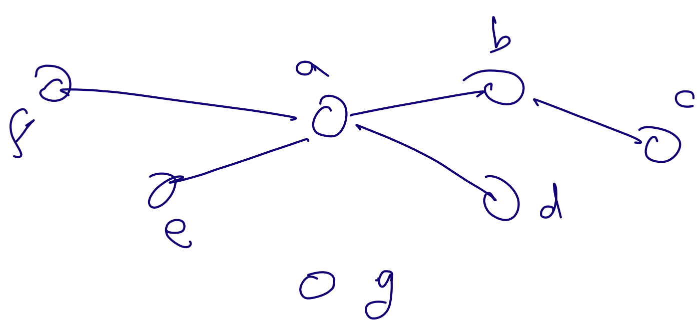
removal of these edges will still result in connected (a,b)

Isolated vertex is in it's own c.c.

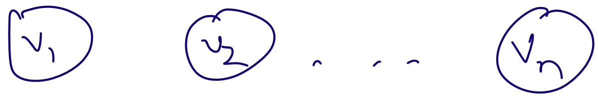
Recap – Path
Connected graph
Connected component
Cycle

degree of a vertex v in an undirected graph is the number of vertices that it is adjacent to.

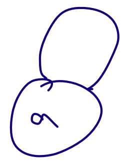
number of edges that are emerging out of v.



deg(a) = 4
deg(c) = 1
deg(d) = 1
deg(b) = 2
deg(g) = 0



Self-loops



$\deg(a) = 2$

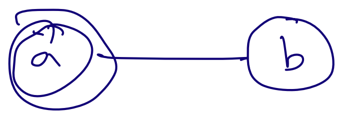
Result Sum of all degrees = 2 * edges.

$$\sum_{v \in V} \deg(v) = \underbrace{2|E|}_{\text{even}}$$

Handshaking lemma.

→ V is people
edge btwn P_1 & P_2 if they shook hands.

Proof



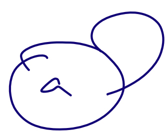
each edge (a, b) counts ______ times
in $\sum_{v \in V} \deg(v) = \deg(v_1) + \deg(v_2) + \ldots + \deg(v_n)$

deg (a) will include a,b edge.
deg (b) also includes it.

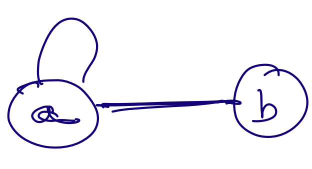
each edge (a, b) counts once ~~for~~ towards deg (a)
and once towards deg (b)

$\therefore \quad 2|E| = \sum \deg(v)$

also note if we have a self loop



this vertex's degree increases by 2 due to self loop



G

$\deg(a) = 3$

$\deg(a) + \deg(b) = 3 + 1 = 4$

Lemma / Corollary - In an undirected graph G
the number of vertices that have an odd degree must be even (try to prove this yourself)

Hand shaking Lemma

" graphviz online "