CIT 5920 10/22 lecture

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Conditional probability

$P(A|B)$ = prob of event A given event B

$$= \frac{P(A \cap B)}{P(B)}$$

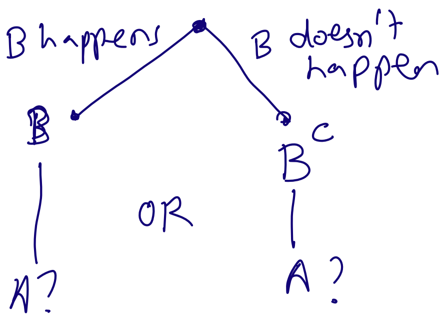
Bayes Theorem

$P(A|B) = P(A \cap B)/P(B)$ ①

$P(B|A) = P(A \cap B)/P(A)$ ②

$$= \frac{P(A|B)\,P(B)}{P(A)} \quad \text{from ①}$$

$$= \frac{P(A|B)\,P(B)}{P(A|B)\,P(B) + P(A|B^c)\,P(B^c)}$$

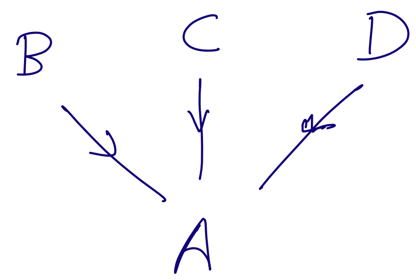


$P(A|B) = P(A \cap B)/P(B)$

$P(A|B)\,P(B) = \frac{P(A \cap B)}{\cancel{P(B)}}\,\cancel{P(B)}$

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)\,P(B) + P(A|B^c)\,P(B^c)}$$

$P(A|B)\,P(B) \rightarrow P(A \cap B)$



$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)\,P(B) + P(A|C)\,P(C) + P(A|D)\,P(D)}$$

Q: 51% of adults in some county (A) are above 35 yrs.

Pick a random ~~person~~ adult in A, what is prob they are > 35 yrs?

Assume some random ~~person~~ adult picked & you are told that they smoke a cigar.

9.5% of adults > 35 smoke cigars

1.7% of adults ≤ 35 smoke cigars.

What is prob that random ~~person~~ adult is > 35 yrs

A - event of > 35 yrs

B - event of smoke cigar

P(A|B) — prob of A "given" B ✓

$9.5\% = 0.095$ = prob of smoking if you are > 35

$= P(B|A)$

$1.7\% = 0.017 = P(B|A^c)$

$P(A) = 57\% = 0.57$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|A^c)\,P(A^c)}$$

$$= \frac{0.095 * 0.57}{0.095 * 0.57 + 0.017 * 0.49}$$

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$= \underline{\underline{0.853}}$

Q: False positives, false negatives

medical test for some disease

T – test returns positive

D – person actually has the disease

bit.ly/CIT5920221

False positive??

↓

test returning positive

$P(T|D^c)$ – test is positive but you are fine!

$P(D^c | T) \stackrel{?}{=} P(T | D^c)$ in general not true

False negative bit.ly/CIT5921022O

↓ what is negative?

$P(T^c | D)$ — test is neg but you have disease.

Q. $D - \frac{5}{1000}$ ppl have a disease

Test with false positive rate 3%

false negative rate 1%

Prob that a person who tested positive actually has disease

T — event of testing positive

$$P(D | T) = \text{by Bayes theorem}$$

$$= \frac{P(T|D)\,P(D)}{P(T|D)\,P(D) + P(T|D^c)\,P(D^c)}$$

(false pos)

(5)

false positive

$P(T | D^c)$ — test is positive but you are fine!

false neg

$P(T^c | D)$ — test is neg but you have disease.

$$P(T|D) = 1 - P(T^c/D) = 1 - \text{false neg rate}$$

Overall takeaway

Bayes th - use it when $P(B|A)$ asked
but only $P(A|B)$ $P(A|B^c)$
are given

Expectation of a random variable

random variable ??
↓
func maps from sample space → $\mathbb{R}$

usually capital letter for random var.

discrete random variables
only map to some finite discrete set of values.

Prob - likelihood of event taking place
r.v. - \$ amount associated with event.

<u>Expectation</u> - practical context
when some random experiment has a payout associated with events occurring
what amount do we <u>expect</u> to earn ?

$$E[X] = \sum_{\substack{\text{all possible} \\ \text{values } t}} P(X = t) * t$$

Q. Roll die. You win 1 $ if die shows 1
2 $ " " 2
etc

What is expected $ amount you earn?
average $ " " "

X – r.v. associated with $ amount for our roll

$$E[X] = 1 * P(X=1) + 2 * P(X=2) + \dots + 6 * P(X=6)$$

$$= 1 * 1/6 + 2 * 1/6 + \dots + 6 * 1/6$$

$$= \underline{3.5}$$