SLIDO #1407232

Lecture 12: Independence & RANDOM VARIABLES

Def.: Two events A and B are independent if the occurrence of one does not affect the prob. of the other.

Mathematically:

$$\mathbb{P}[A \cap B] = \mathbb{P}[A] \times \mathbb{P}[B]$$

will show us how to combine probabilities

Peer pressure

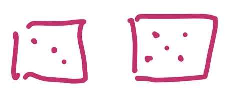
$$\mathbb{P}[\text{I want to do something}] \neq \mathbb{P}[\text{I want to do something} \mid \text{other people do it}]$$

Criteria for Independence

- $\mathbb{P}[A \mid B] = \mathbb{P}[A]$
- $\mathbb{P}[B \mid A] = \mathbb{P}[B]$

reads as "the probability of (event) B happening, knowing that (event) A has happened, is equal to the prob. of (event) B happening"

EX1. Two dice rolled. Are the events "the 1st die rolls a 3" and "the 2nd die rolls a 5" independent?



Event is (3,5)

Sample Space is $\{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$
$= \{(1,1), (1,2), (1,3) \dots (2,1) \dots (6,6)\}$

Answer: $\mathbb{P}[\text{1st die rolls a 3}] = \frac{1}{6}$

$\mathbb{P}[\text{2nd die rolls a 5}] = \frac{1}{6}$

$\mathbb{P}[\text{1st die rolls a 3 AND 2nd die rolls a 5}] = \frac{1}{36}$

since $\boxed{\frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6}}$ the events are independent

Follow-up: Are the events "the 1st die rolls a 3" and "the 1st die rolls a 5"

How do we "break" the example?

Answer: $\mathbb{P}[\text{1st die rolls a 3}] = \frac{1}{6}$

$\mathbb{P}[\text{1st die rolls a 5}] = \frac{1}{6}$ ← impossible conjunction of events

$\mathbb{P}[\text{1st die rolls a 3 and 1st die rolls a 5}] = 0$

since $0 \neq \frac{1}{6} \cdot \frac{1}{6}$ the events are dependent.

EX2. Dependent card draws.

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Are the events "drawing a spade ♠" and "drawing an Ace" independent when drawing a card from a deck

$\mathbb{P}[\text{drawing a spade}] = \frac{13}{52} = \frac{1}{4}$ ← one out of the 4 colors

$\mathbb{P}[\text{drawing an Ace}] = \frac{4}{52}$

$\mathbb{P}[\text{Ace of Spades}] = \frac{1}{52}$

Since $\frac{1}{52} \neq \frac{13}{52} \times \frac{4}{52}$ then the events are dependent.

RANDOM VARIABLES (RV)

Def. A random variable is a function that assigns a numerical value to each outcome in the sample space

Notation: often uppercase letters, X, Y, Z

Def. A discrete random variable is a RV that can take on a countable number of values

Def. A probability Mass Function (PMF) is a function that gives the probability that a discrete Random variable is equal to some value.

$$\mathbb{P}[X = x] = p(x)$$

EX1. Sum of dice rolls

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- Experiment: Roll two fair 6-sided die
- Random Variable: X = sum of the numbers on the two die
- Possible Values: X can take any value between 2 and 12
  ($2 = 1+1$, $12 = 6+6$)
- PMF $\mathbb{P}[X=2] = \frac{1}{36}$ because (1,1)

  $\mathbb{P}[X=3] = \frac{2}{36}$ because (1,2), (2,1)

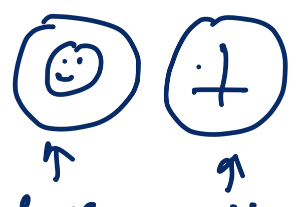
  $\mathbb{P}[X=4] = \frac{3}{36}$ because (1,3), (2,2), (3,1)

  ⋮ all the possible ways to add up to X using two die roll between 1 and 6

  $\mathbb{P}[X=12] = \frac{1}{36}$ because (6,6)

EX2. Number of Coin Flips

coin → HEADS 1/2 / TAILS 1/2



face tails

- Experiment: Flip a coin 5 times
- Random Variable: X = number of times it is TAILS
- Possible Values: 0, 1, 2, 3, 4, 5
- PMF: $\mathbb{P}[X=0] = \left(\frac{1}{2}\right)^5$ ← contribution from coins falling as HEADS

  $\mathbb{P}[X=1] = 5\,\frac{1}{2} \cdot \left(\frac{1}{2}\right)^4$

  contribution from TAILS

  $\mathbb{P}[X=2] = \binom{5}{2}\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$

  ⋮

  $\mathbb{P}[X=k] = \binom{5}{k}\left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{5-k}$

  $1 \leq k \leq 5$