Lecture 10: Discrete Probability Continued

Notation remark

Consider the UNIFORM DICE ROLL

because everything is equally likely under uniform probability

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

E_1 : event "dice is odd"
$E_1 = \{1, 3, 5\} \subset S$

E_2 : event "dice is even"
$E_2 = \{2, 4, 6\} \subset S$

E_3 : event "dice is prime"
$E_3 = \{2, 3, 5\} \subset S$

E_4 : event "dice is not prime"
$E_4 = \{1, 4, 6\} \subset S$

E_5 : event "dice can be expressed as 2^N with N integer"
$E_5 = \{1, 2, 4\} \subset S$
$2^0 \quad 2^1 \quad 2^2$

E_6 : event "dice is a perfect square (1×1, 2×2, 3×3...)"
$E_6 = \{1, 4\} \subset S$

E_7 : event "dice can be divided by 3"
$E_7 = \{3, 6\} \subset S$

$\mathbb{P}[E_1] = \frac{3}{6} = \left[\frac{1}{2} = 0.5\right]$

$\mathbb{P}[E_2] = \frac{3}{6}$

$\mathbb{P}[E_3] = \frac{3}{6}$

$\mathbb{P}[E_4] = \frac{3}{6}$

$\mathbb{P}[E_5] = \frac{3}{6}$

$\mathbb{P}[E_6] = \frac{2}{6} \left[= \frac{1}{3}\right]$

$\mathbb{P}[E_7] = \frac{2}{6}$

$\mathbb{P}[E_i]$ = probability that E_i occurs $= \frac{|E_i|}{|S|}$

$|E_i|$ ← POSITIVE OUTCOMES (size of event)

$|S|$ ← total outcomes (size of sample space)

Exercise. What is an event that has probability ...

- 5/6?

E_8: "dice is greater or equal than 2"
$E_8 = \{2, 3, 4, 5, 6\}$ $\mathbb{P}[E_8] = \frac{5}{6}$

- 1/6 ?

E_9: "dice is divisible by 5"
$E_9 = \{5\}$ $\mathbb{P}[E_9] = \frac{1}{6}$

NON-UNIFORM DISTRIBUTION

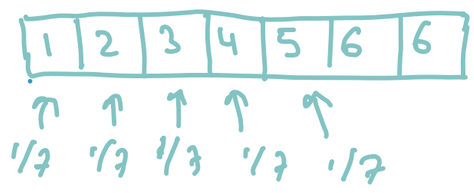
Def: A non-uniform distribution is a proba. distribution where outcomes have different probabilities.

This is used/useful when certain outcomes are more likely than others.

EX 1. A loaded die ("loaded" = {unfair / cheating})

Suppose a 6-sided die is weighted (= modified) so that the number 6 is 2-times more likely to come up than other numbers.

- for $k \in \{1, 2, 3, 4, 5\}$ $\mathbb{P}[\text{dice is } k] = \frac{1}{7}$
- for $k = 6$ $\mathbb{P}[\text{dice is } 6] = \frac{2}{7}$

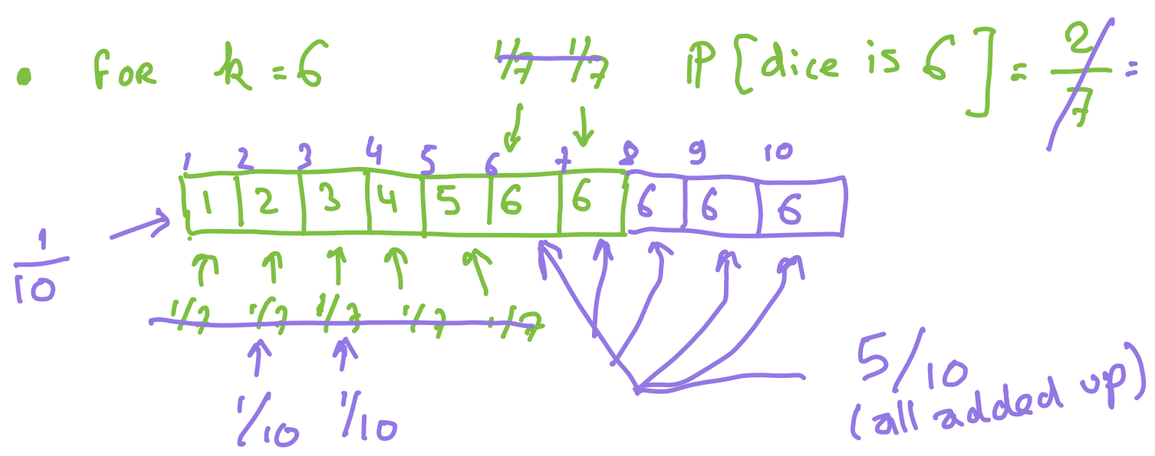


Suppose a 6-sided die is weighted (= modified) so that the number 6 is 5-times more likely to come up than other numbers.

- for $k \in \{1, 2, 3, 4, 5\}$ $\mathbb{P}[\text{dice is } k] = \frac{1}{7} = \frac{1}{10}$
- for $k = 6$ $\mathbb{P}[\text{dice is } 6] = \frac{2}{7} = \frac{5}{10}$

} also represents a distribution (this is the preferred)

} this represents a non-uniform distribution



Suppose a 6-sided die is weighted (= modified) so that the number 6 is ~~4~~ 2-times more likely to come up than other numbers. and 5 is 3-times more likely.

⇒ What is the probability distribution?