Lecture 9B : Introduction to DISCRETE probabilities

Definitions of Experiment, Sample Space and Event

- **Experiment** : A procedure that yields one out of several possible outcomes. It's an action or process whose outcome is uncertain.
- **Sample Space (S)**: The set of all possible outcomes of an experiment.
- **Event**: A subset of the sample space. An event occurs if the outcome of the experiment is an element of the event set.

Note: An outcome is a single possible result of an experiment, while an event can consist of one or more outcomes.

EXAMPLES

EX1. Rolling Dice

- **Experiment**: Rolling two 6-sided dice one RED and one BLUE
- **Sample Space**:
  - all possible ordered pairs (x, y) where x is the outcome on the red die and y is the outcome on the blue die
  - total outcomes $6 \times 6 = 36$
- **Event**: Getting a sum of 7

$$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

EX2. Drawing Mahjong Tiles

- **Experiment**: drawing a single tile from a complete set of Mahjong tiles
- **Sample Space**: all 136 tiles in the traditional Mahjong set
- **Event**: drawing a DRAGON tile

  E: the set of 12 dragon tiles (Red, green, white).

PROBABILITY DISTRIBUTION

A probability distribution assigns a probability to each outcome in the sample space S.

PROPERTIES of $P(s)$

- $0 \leq P(s) \leq 1$ for every $s \in S$
- $\sum_{s \in S} P(s) = 1$

The probability of an event is the sum of the probabilities of the outcomes in E:

$$P(E) = \sum_{s \in E} P(s)$$

UNIFORM PROB DISTRIBUTION

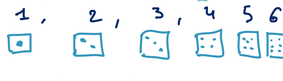
Def A distribution where every outcome in the sample space is equally likely

Then the prob. of each outcome:

if $|S| = n$

then $P(s) = \frac{1}{n}$ for all $s \in S$

EX1. Fair Die Roll [singular → die, plural → dice]

- **Sample Space** $S = \{1, 2, 3, 4, 5, 6\}$ 
- **Probability of Each Outcome is**

$$P(s) = \frac{1}{|S|} = \frac{1}{6}$$

EX2. Dealing tiles in Xiangqi (Chinese Chess)

- **Experiment**: Randomly selecting a piece from all the pieces used
- **Sample Space**: all 32 pieces (16 red and 16 black)
- **Probability under uniform distribution**

$$P(s) = \frac{1}{32}.$$