Penn

# CIT 5920–Mathematical Foundations of Computer Science
# Recitation 4: *Sets, set operations and relations*

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Please write a brief explanation for each of these questions. This is also your opportunity to ask the TAs how much to write in these types of questions.

**Exercise 1**
In how many ways can we form 4 letter words using 4 letters from the word "ADDRESS"? To clarify, assume each of those letters was a block (think about Scrabble or Words with Friends if you have played those games). This question says you have only 7 letter blocks and you can only use those blocks. A word like "ADRE" is possible but a word like "ADRA" is not possible because you have only 1 "A".

**Exercise 2**
There are 32 first year MCITs and as part of 5910 recitation they are told to randomly pair up and do pair programming. How many different pairings are possible? Note that we are counting the distinct pairings and not the number of ways to pick one pair.
For example, with 4 people - $A$, $B$, $C$, and $D$, we could have $A$ with $B$ or $A$ with $C$ or $A$ with $D$, and in each case, the remaining two are in a pair. These are the possibilities, so the answer is 3.
Hint: The answer is not $\binom{32}{3}$.

**Exercise 3**
In how many ways can 6 people be seated around a circular table? Be careful about not over-counting equivalent arrangements.

**Exercise 4**
How many solutions are there to this inequality $x_1 + x_2 + x_3 \leqslant 12$, but under each of the following constraints:

A. Each of the $x_i$ must be strictly positive integers.

B. Each of the $x_i$ must be strictly positive integers, and $x_1 + x_2 \geqslant 5$.

C. Each of the $x_i$ must be strictly positive integers, and $x_3 < 5$.

**Exercise 5**
How many 18 bit numbers exist that have exactly 8 0s and 10 1s if every zero must immediately be followed by a 1?

**Exercise 6**
Suppose a person rolls $n$ identical 6 sided dice and they roll them all at once. How many distinct throws are possible?

 

For clarity consider a scenario where there are 7 dice. Then we will consider 1122331 the same as 1112233. Also the same as 1123231.

### Exercise 7

In how many ways can 10 different rings be distributed among the fingers of one hand? It is assumed that any finger can hold all rings.

### Exercise 8

You are an environmental scientist working to protect a delicate ecosystem from pollution. You have 1000 samples of water from different parts of a river, and one is contaminated with a harmful chemical. Using 10 test strips, you need to identify the contaminated sample to prevent ecological damage and protect biodiversity. A single drop of contaminated water will turn the test strip positive permanently. You can put any number of drops on a test strip at once and you can reuse a test strip as many times as you'd like (as long as the results are negative). However, you can only run tests once per day and it takes seven days to return a result. How would you figure out the contaminated sample in as few days as possible?

 