Lecture 9

Quiz: How many functions can be formed from a set with 3 elements to a set with 4 elements?

- What is a function?

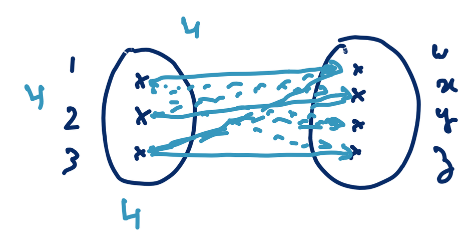
Sets $\xrightarrow{\text{pairs}}$ Cartesian Products $\xrightarrow{\text{subsets}}$ relations $\xrightarrow{?}$ function

single pair for each x in the domain

$A = \{1, 2, 3\}$ $\quad |A| = 3$

$B = \{w, x, y, z\}$ $\quad |B| = 4$

a function maps each element of A to a single element in B



to "create" a function, we must find a way to choose the arrows from A to B

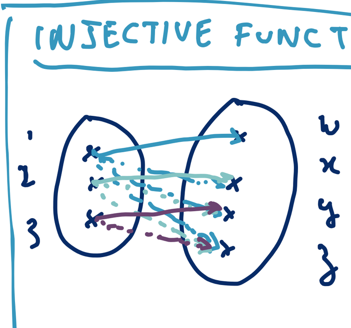
to count the functions, we need to count all possible ways of placing those arrows

$4 \times 4 \times 4 = 4^3$ total functions

why products? (because independant) Rachel

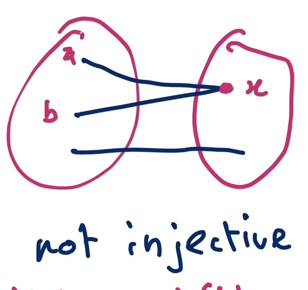
Jom: what about injective

INJECTIVE FUNCTIONS COUNTING



$(1, w)$

$4 \times 3 \times 2 = \boxed{24}$

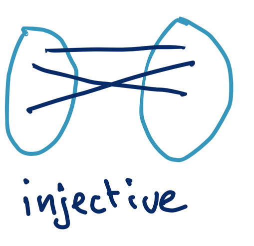


not injective

$f(a) = x = f(b)$

but $a \neq b$

one element on B has two arrows incoming $\rightarrow$ X

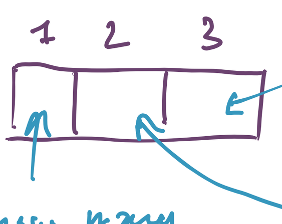


injective

Quiz: How many ways are there to arrange 3 people in a line if there are 4 people to choose from?

P = { Jérémie, Arvind, Kyrie, Harry }

line $\rightarrow$



$4 \times 3 \times 2 = 24$ different ways

how many options could go here: 4

3 because one person is already in front

2

PERMUTATIONS

A permutation refers to the number of ways to arrange a set of specific objects in a specific order, where all objects are distinct and order matters

- Total permutations of n distinct objects: The number of ways to arrange n distinct objects in sequence is $n!$ (pronounced "n factorial") defined as:

$$n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

- Permutation of R objects from n distinct objects: The number of ways to arrange r objects out of n distinct objects:

$$P(n, R) = n \times (n-1) \times \dots \times (n - R + 1) = \frac{n!}{(n-R)!}$$

⚠ Important:

- ORDER MATTERS: Changing the order of objects creates a different permutation
- DISTINCT OBJECTS: All objects are distinct

EXAMPLES

Example 1: How many ways can you arrange 4 books on a shelf?

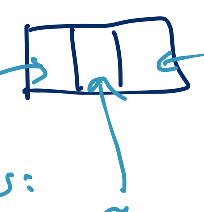
$\hookrightarrow 4! = 4 \times 3 \times 2 \times 1 = 24$

Example 2: In a competition, with 5 athletes, how many ways to assign medals — gold, silver, bronze?

$\hookrightarrow 5 \times 4 \times 3 = P(5, 3)$

Example 3: How many 3-digit PIN code can be created using the digits 0-9 if no digit is repeated and the first digit cannot be zero?

$9 \times 9 \times 8$



8 options: 0-9 minus the two chosen previously

9 options: 1-9

9 options: 0-9 minus the one chosen in first spot

COMBINATIONS

A combination refers to the number of ways to select a subset of objects from a larger set where order does not matter, and all objects are distinct

- Combination of R objects chosen from N distinct objects

$$C(n, R) = \binom{n}{R} = \boxed{\frac{n!}{(n-R)!\, R!}} = \frac{P(n, R)}{R!}$$

$P(n, R)$ ← permutation (where order matters)

R! ← "orderings"

⚠ Important

- ORDER DOES NOT MATTER: Changing the order of the selected objects does not create a new combination
- DISTINCT OBJECTS: All objects are different from each other

EXAMPLES

- **Example 1**: From a group of 7 students, how many ways can you choose a committee of 3 students?

$\hookrightarrow C(7, 3) = \binom{7}{3} = \frac{7!}{(7-3)!\, 3!} = \frac{7!}{4!\, 3!} = \frac{7 \times \cancel{6} \times 5}{\cancel{3!}} = 7 \times 5 = 35$

$= \frac{7!}{4!\, 3!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \times 3 \cdot 2 \cdot 1} = \frac{7 \cdot \cancel{6} \cdot 5}{\cancel{3} \cdot \cancel{2} \cdot 1} = 7 \cdot 5 = 35$

- **Example 2**: In a lottery game, you must choose 6 numbers out of 49. How many different sets of numbers can you choose?

$\hookrightarrow C(49, 6) = \binom{49}{6} = \frac{49!}{(49-6)!\, 6!} = \frac{49!}{43!\, 6!} = \frac{49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

- **Example 3**: An ice-cream shop offers 10 flavors. How many ways can you choose 2 flavors for a double-scoop cone if the order of scoops doesn't matter (and can't choose two of the same)

$\hookrightarrow C(10, 2) = \binom{10}{2} = \frac{10!}{(10-2)!\, 2!} = \frac{10!}{8!\, 2!} = \frac{10 \cdot 9}{2!} = 5 \times 9 = 45$