CIT 5920
Recitation 1

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Penn Engineering

Overview for Today

- Logistics
- Brief review
- Question 1 (3 minutes)
- Question 2 (3 minutes)
- Question 3 (3 minutes)
- Question 4 (6 minutes)
- Question 5 (6 minutes)
- Question 6 (6 minutes)
- Question 7 (5 minutes)
- Question 8 (5 minutes)

Logistics

- TA Office Hours are still TBD
  - PLEASE FILL OUT **WHEN2MEET** BY END OF DAY
- Canvas is now live!
- First homework will be released this evening

New notions

- Cartesian product
- Powerset
- Cardinality

Question 1a

What is $\mathbb{R} - \mathbb{Q}$?

What is $\mathbb{Q} \cap \mathbb{Z}$?

Answer 1

$\mathbb{R}$ = Real numbers

$\mathbb{I}$ = Imaginary numbers

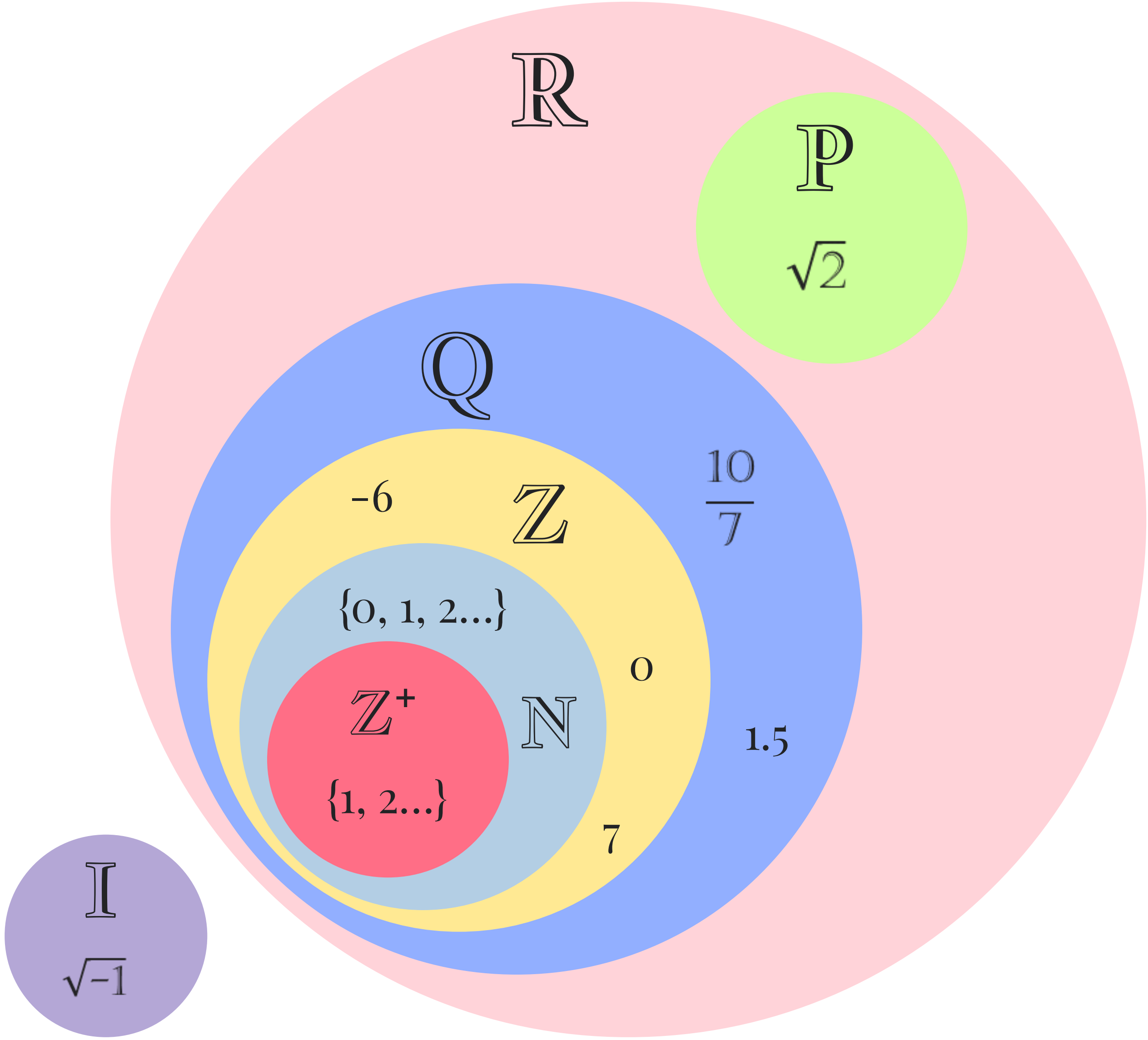
$\mathbb{Q}$ = Rational numbers

$\mathbb{P}$ = Irrational numbers

$\mathbb{Z}$ = Integers

$\mathbb{W}$ = Whole numbers

$\mathbb{N}$ = Natural numbers



Answer 1 (cont.)

What is $\mathbb{R} - \mathbb{Q}$?

Solution: $\mathbb{P}$ (irrationals)

What is $\mathbb{Q} \cap \mathbb{Z}$?

Solution: $\mathbb{Z}$

Cartesian product

A x B, called the cartesian product of A and B consists of ordered pairs of the form (a,b) where a $\in$ A and b $\in$ B.

Set A	Set B	A x B =
2	x	(2,x)
5	y	(2,y)
		(5,x)
		(5,y)

Question 1b (a bit harder with new stuff)

What is A x B when A= {a,b,c} and B = {3,5}

What is $\mathbb{R} \times \emptyset$?

Answer 1 (cont.)

A x B = {(a,3) , (a,5) , (b,3), (b,5), (c,3), (c,5)}

What is $\mathbb{R} \times \emptyset$?

Solution: $\emptyset$

Cardinality

The number of elements in a finite set is called the cardinality of that set. A is a set, the cardinality is denoted by |A|.

Ex: A = {1,{3,4,5},5}. *|A| = 3*

Power Sets

The power set of a set A is the set of all subsets of A and is denoted P(A).

Remember: P(A) always include both the empty set and A itself.

Ex: A = {x,y}

P(A) = {{}, {x}, {y}, {x,y}}

Question 2

If $A \subseteq B$ does this mean $P(A) \subseteq P(B)$.

Either justify this statement (no formal proof required) or provide a counterexample.

What is P(F) when F = {1,2,5}?

What is the cardinality of P(F)?

What is the cardinality of a powerset of set A of length n?

Answer 2

Solution:

This statement is true.

Let $x \in P(A)$. By definition of a power set, $x \subseteq A$. If $x \subseteq A$, then $x \subseteq B$. Again, by definition of power set, $x \in P(B)$. Thus, $P(A) \subseteq P(B)$.

Note: This solution is more formal than required at this point in the course; it's okay if your answers are written less mathematically until formal proofs are covered later on.

P(F) = { {}, {1}, {2}, {5}, {1,2}, {1,5}, {2,5}, {1,2,5} }

|P(F)| = 8

Question 3

What is $P(P(\emptyset))$?

This question is purely to test your understanding

Answer 3

Solution:

$P(\emptyset) = \{\emptyset\}$

$P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$

Question 4

For the sets A, B, C as defined below

$A = \{1, 2, 5, 8\}$ $B = \{3, 4, 1, 2, 7\}$ $C = \{3, 2\}$

what is

$A \cup B$, $A \cap B$, $A \times C$, $C \times A$, $A - B$, $(A - B) \cap C$

Answer 4

Solution:

$$A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$$
$$A \cap B = \{1, 2\}$$
$$A \times C = \{\{1, 3\}, \{2, 3\}, \{5, 3\}, \{8, 3\}, \{1, 2\}, \{2, 2\}, \{5, 2\}, \{8, 2\}\}$$
$$C \times A = \{\{3, 1\}, \{3, 2\}, \{3, 5\}, \{3, 8\}, \{2, 1\}, \{2, 2\}, \{2, 5\}, \{2, 8\}\}$$
$$A - B = \{5, 8\}$$
$$(A - B) \cap C = \emptyset$$

These should be parenthesis ^

Question 5

List the elements of these sets explicitly

$\{x | x \in \mathbb{Z} \text{ and } x^2 < 5\}$

$\{x | x \in \mathbb{Q} \text{ and } x = 2^n \text{ for some } n \in \mathbb{Z}\}$

Answer 5

$\{x | x \in \mathbb{Z} \text{ and } x^2 < 5\}$

Solution: $\{-2, -1, 0, 1, 2\}$

$\{x | x \in \mathbb{Q} \text{ and } x = 2^n \text{ for some } n \in \mathbb{Z}\}$

Solution: $\{\ldots \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4 \ldots\}$

Question 6

Write the following sets in set builder form by identifying some property that all the elements share. Try and write these in the most mathematical manner possible.

- $\{1, 4, 9, \ldots, 100\}$
- $\{3, 7, 11, 15, \ldots\}$
- $\{-100, -99, -98, \ldots, 5\}$
- $\{5, 10, 15, 20, 25, 30\}$
- $\{0.5, 0.125, 0.0625, 0.25\}$

Answer 6

Solution:

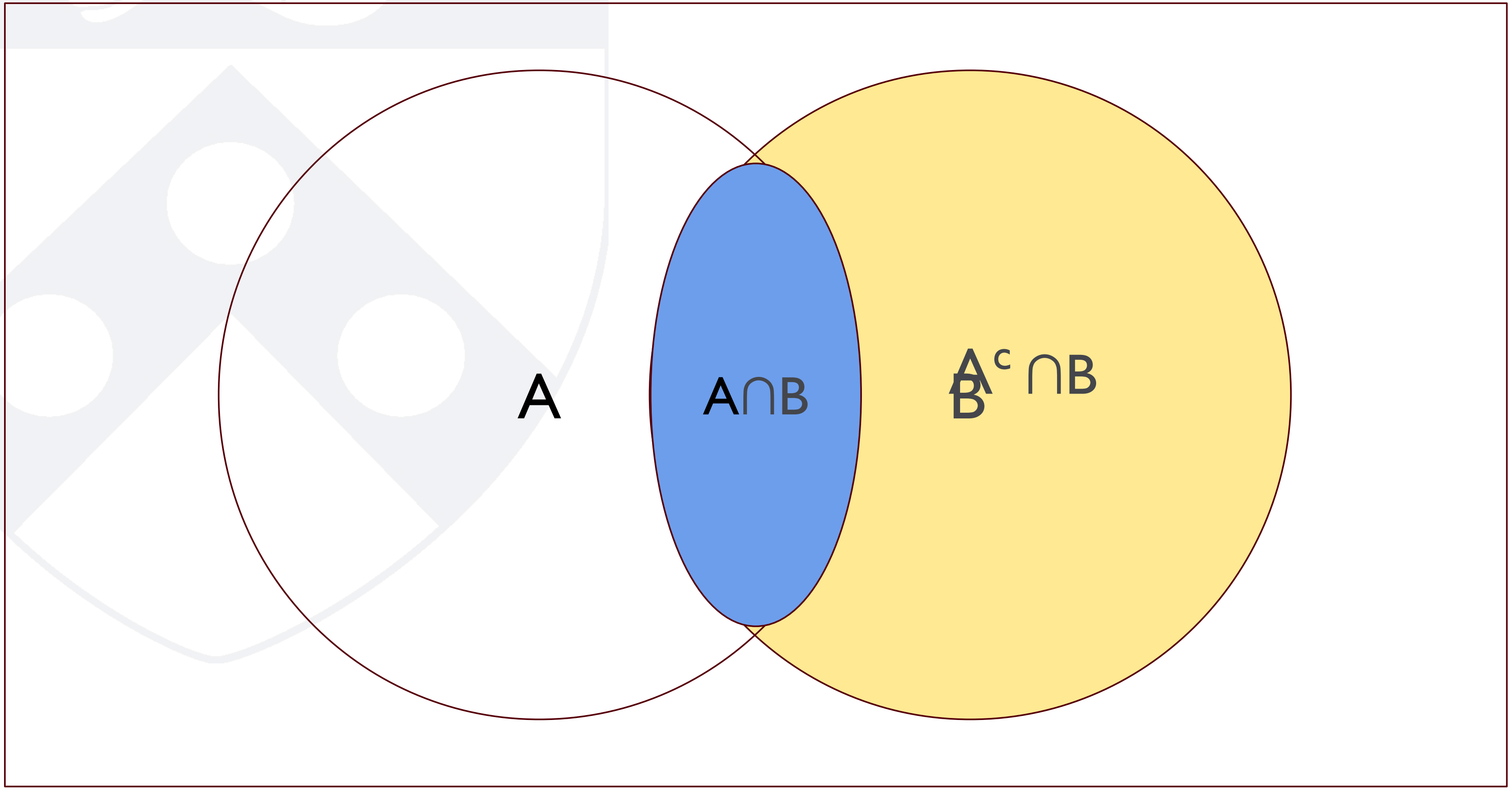
- $\{1, 4, 9, \ldots, 100\}$
  $a = \{x|x = n^2, n \in \mathbb{Z} \text{ and } -10 \leq n \leq 10 \text{ and } n \neq 0\}$
- $\{-100, -99, -98, \ldots, 5\}$
  $b = \{x|x \in \mathbb{Z} \text{ and } -100 \leq x \leq 5\}$
- $\{3, 7, 11, 15, \ldots\}$
  $c = \{x|x = 4n + 3, n \in \mathbb{N}\}$
- $\{5, 10, 15, 20, 25, 30\}$
  $d = \{x|x = 5n, n \in \mathbb{N} \text{ and } 1 \leq n \leq 6\}$
- $\{0.5, 0.125, 0.0625, 0.25\}$
  $e = \{x|x = \frac{1}{2^n}, n \in \mathbb{N} \text{ and } 1 \leq n \leq 4\}$

Question 7

7. Show that $(A \cap B) \cup (A^c \cap B) = B$.

Use Venn Diagrams and colors (or textures/fillings) to show this.

Solution



Quick reminder of what properties are

What is commutative?

What is distributive?

if you are going to do the next question

Answer 7

Solution:

$$
\begin{aligned}
(A \cap B) \cup (A^c \cap B) &= (B \cap A) \cup (B \cap A^c) && (1)\\
&= B \cap (A^c \cup A) && (2)\\
&= B \cap U && (3)\\
&= B && (4)
\end{aligned}
$$

1. Intersection is commutative
2. Distributive property
3. The union of A and A^c is the universe
4. The intersection of B and the universe is B

Question 8

Show that $(A - C) \cap (C - B) = \emptyset$. No using Venn diagrams. Use set identities. Recall that set difference can be written in terms of an intersection.

Answer 8

Solution:

$\because$ A - C = A $\cap$ C^c

$\because$ C - B = C $\cap$ B^c

$\therefore (A - C) \cap (C - B)$

$= A \cap C^c \cap C \cap B^c$

$= A \cap (C^c \cap C) \cap B^c$

$= A \cap \emptyset \cap B^c$

$= \emptyset$

See you next week!