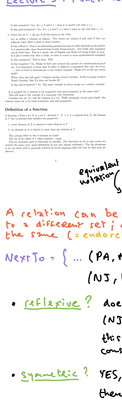
Lecture 5 : Functions



First another example of a relation.

$U = \{$ all states in the USA $\}$

PA
Pennsylvania $\in U$

New Jersey $\in U$
NJ

NextTo $\subseteq U \times U$

Let NextTo be the relation: two states are related if and only if they are adjacent (they share a border)

equivalent notations

$(PA, NJ) \in$ NextTo

PA NextTo NJ ← infix notation

A Formal definitions

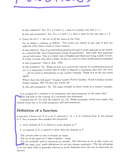
A relation can be from one set to a different set; or from set to the same (= endorelation).

NextTo = $\{ \ldots (PA, NJ), (NJ, PA), (NY, NJ) (NJ, DE), \ldots \}$

- reflexive? does it contain pairs (x,x) (NJ, NJ) or (PA, PA)
  this depends on whether you considad a state bordaral itself
  PA NextTo NJ
- symmetric? YES, because if PA bordaes NJ then this relation is recipAocal and NJ boardaes PA
  NJ NextTo PA
- transitive? NO. Example: $(PA,NJ) \in$ NextTo, $(NJ,NY) \in$ NextTo, yet we have $(PA,NY) \notin$ NextTo (equiv. notations) PA ~~NextTo~~ NY
- anti-symmetric? the relation would be antisymmetric only if $(NJ,PA) \in$ NextTo would imply that $NJ = PA$ which is clearly not true.

Sets —make pairs→ Cartesian Products —subset→ Relation ……? Function

FUNCTIONS



A function f from a set X to a set Y, denoted

$f : X \longrightarrow Y$

is a relation from

X : the domain, input [set], source

to

Y : the co-domain, output [set], range, target ← alternate terminology

that satisfies two properties:

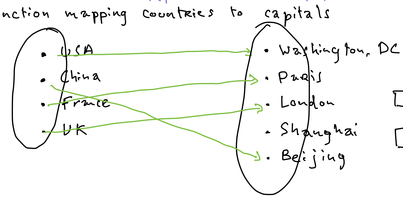
1. Every element of X is related to some element of Y — ALL
2. No element of X is related to more than one element of Y — NO COLLISIONS

= 

EXAMPLES

domain = input; co-domain = output

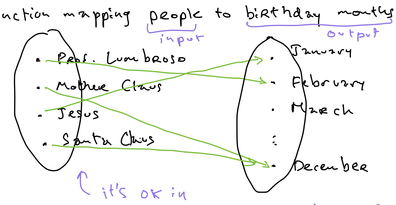
- Function mapping countries to capitals



✓ ALL — because otherwise the function would "undefined" for certain inputs

✓ NO COLLISIONS

- Function mapping people (input) to birthday months (output)



it's OK in a function for several inputs to have the same output

$f: A \longrightarrow B$

$g: B \longrightarrow C$

$g(f(x)) : A \longrightarrow C$ composed function

CEILING & FLOOR functions

Ceiling function $\lceil x \rceil$ is the smallest integer y such that $y \geq x$

E.g. $\lceil 4.51 \rceil = 5$ $\lceil -1.27 \rceil = -1$ $\lceil 0 \rceil = 0$

Floor function $\lfloor x \rfloor$ is the largest integer y such that $y \leq x$

E.g.: $\lfloor 1.2 \rfloor = 1$ $\lfloor -2.7 \rfloor = -3$ $\lfloor 0 \rfloor = 0$

Properties of functions: INJECTIVE (one-one or "one-to-one") SURJECTIVE ("onto") BIJECTIVE

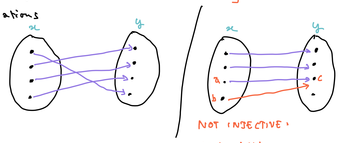
INJECTIVE FUNCTION OR ONE-ONE

A function f is injective if and only if

$f(a) = f(b)$ implies $a = b$

must be REVERSIBLE

Representations



NOT INJECTIVE:

$f(a) = f(b) = c$

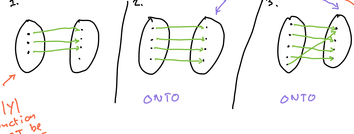
BUT $a \neq b$

SURJECTIVE FUNCTION OR ONTO

A function $f: X \to Y$ is said to be surjective if the entire codomain Y is covered:

if for every $y \in Y$ there is some $x \in X$ such that $f(x) = y$

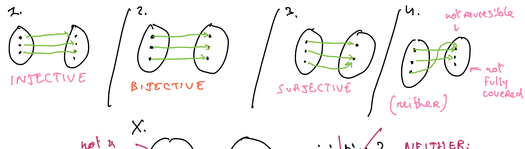
every dot in target must receive an arrow

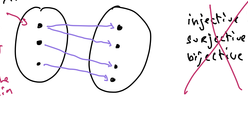


if $|X| < |Y|$ the function CANNOT be SURJECTIVE

BIJECTIVE FUNCTION

A function $f: X \to Y$ is bijective if and only if it is both injective AND surjective.





injective? ~~NEITHER:~~
surjective? ~~IT'S NOT~~
bijective? ~~A FUNCTION~~