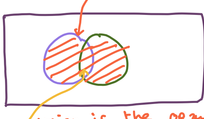
Lecture 3 : Set operations

* **UNION** $A \cup B$ — "cup" or "union"

the set formed from all the elements in A and all the elements in B

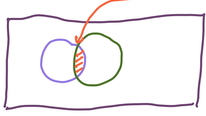


union is the orange shaded area

overlap = repetitions / elements in both sets are counted once

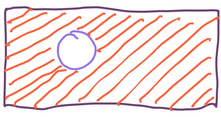
* **INTERSECTION** $A \cap B$ — "cap" or "intersect"

the set formed from elements BOTH in A and B



* **COMPLEMENT** $\bar{A}$ (or sometimes A')

is the set of all elements (of the universe $\mathcal{U}$) that are NOT in A



universe

$\bar{A} = \mathcal{U} \setminus A$

$\bar{\bar{A}} = A$

All can be written formally with set builder notation

$A \cup B = \{x \in \mathcal{U} \mid x \in A \text{ OR } x \in B\}$

$A \cap B = \{x \in \mathcal{U} \mid x \in A \text{ AND } x \in B\}$

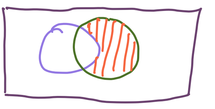
$\bar{A} = \{x \in \mathcal{U} \mid x \notin A\}$

not $x \in A$

$!(x \in A)$

} equiv. options

* **DIFFERENCE** $B \setminus A$: elements in B that are not in A



Properties of Set Operations

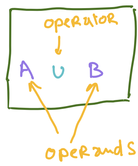
commutative
associative
distributive ⚠

Commutative You can {swap / exchange} {flip / operands} around the operator

$A \cup B = B \cup A$ so $\cup$ is commutative

$A \cap B = B \cap A$ so $\cap$ is commutative

Associative When grouping operations with parentheses, you can change the order of the parenthes, when dealing with a sequence of the SAME OPERATION



$(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$

Distributive (this is the only property where we can mix operator types)

You can factor or distribute over this operator

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

"intersection is distributive over union"

What does it mean to not associative

$1 - 5 - 5 \neq 1 - (5 - 5)$

$-9 \qquad\qquad 1$

} not associative

$1 + 5 + 5 \qquad 1 + (5 + 5)$

$11 \qquad\qquad 11$

} associative

$A \cap \varnothing = \varnothing$ — $\varnothing$ is dominant for $\cap$

$A \cup \mathcal{U} = \mathcal{U}$ — $\mathcal{U}$ is dominant for $\cup$

$A \cap \mathcal{U} = A$ — $\mathcal{U}$ is neutral for $\cap$

$A \cup \varnothing = A$ — $\varnothing$ is neutral for $\cup$

DE MORGAN'S LAW

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

flip