Lecture 2: Set operations

Element of and subset of

A very common confusion is that between the $\in$ and $\subseteq$. Here is an attempt at clarifying that via an example (the zybook has some too).

Consider the set $S = \{CIT, \pi, \{\text{apples}, \text{bananas}\}\}$
What are the elements of this set?
An element of the set is a single item inside the set. So the elements are

1. CIT
2. π
3. {apples, bananas}

And YES, one of the elements is this weird set of some fruit.
What are the subsets of this set?
This questions becomes easier to answer now that we have answered the elements question. Subsets after all are made by collecting none, some or all of these elements and putting them between { and }, that is, making a set out of them.
So here are all the subsets

1. $\emptyset$
2. $\{CIT\}$
3. $\{\pi\}$
4. $\{\{apples, bananas\}\}$ - this one is slightly tricky, but just think of the apples and bananas set as this one single entity.
5. $\{CIT, \pi\}$
6. $\{\pi, \{\text{apples}, \text{bananas}\}\}$
7. $\{CIT, \{\text{apples}, \text{bananas}\}\}$
8. $\{CIT, \pi, \{\text{apples}, \text{bananas}\}\}$

Venn diagrams

A Venn diagram is just a pictorial representation of a set.
Generally speaking, we draw one big box for the universe. The universe can be different depending upon the context being used. For instance, with the numbers (especially in this course), it makes sense to think of the universe as $\mathbb{R}$.

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⚠ **Common confusion**

item $\in$ Set "belongs to"

Subset $\subset$ Set "is a subset of" (Subset: must be a set)

Example: $X = \{1, \{7\}\}$
- X is a set
- 1: an element that is the integer 1
- $\{7\}$: an element that is itself a set containing 7

Which statements are true:

- $1 \in X$ (true)
- ~~$7 \in X$~~ +
- ~~$X \subset 1$~~ *
- ~~$1 \subset X$~~ *
- ~~$7 \subset X$~~ *
- ~~$X \subset \{7\}$~~ +
- ~~$\{1\} \in X$~~ +
- $\{7\} \in X$ (true)
- $\{1\} \subset X$ (true)
- ~~$\{7\} \subset X$~~ +

* syntactic problem: it doesn't make sense
+ incorrect statement: it false

→ true because: $1 \in \{1\}$ and $1 \in X$

Questions

* is $\{\{7\}\}$ a subset of X? YES $\{\{7\}\} \subset X$
* $\{7\} \subset \{7\}$ should be read "the set containing 7 is a subset of the set containing 7"
"7 is a subset of the set containing 7" is an syntactically wrong statement

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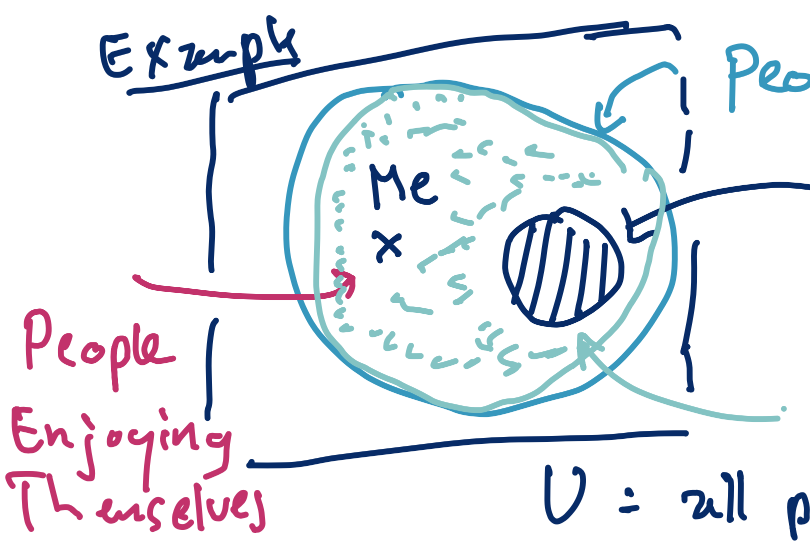
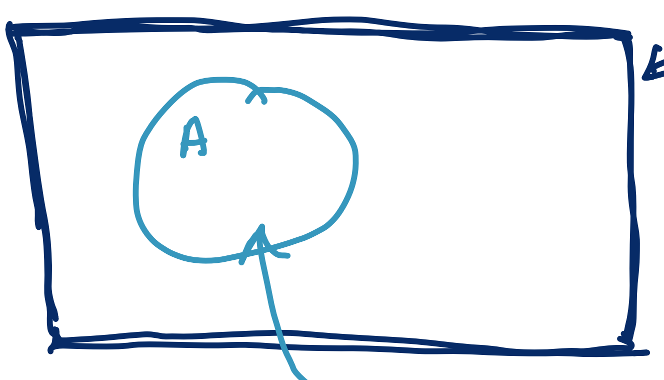
Venn diagrams = pictorial representation of a set

Example



People in this class
People asking questions on Slido
People answering the polls

U: all people or all people at Penn

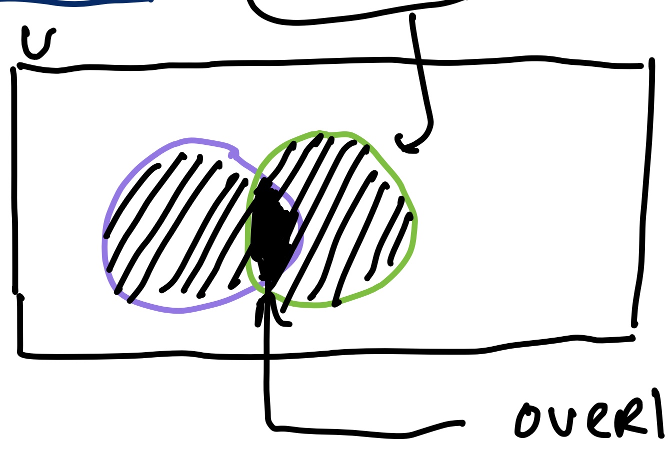


frame is often U = the universe

the set A within the universe
$A \subseteq U$

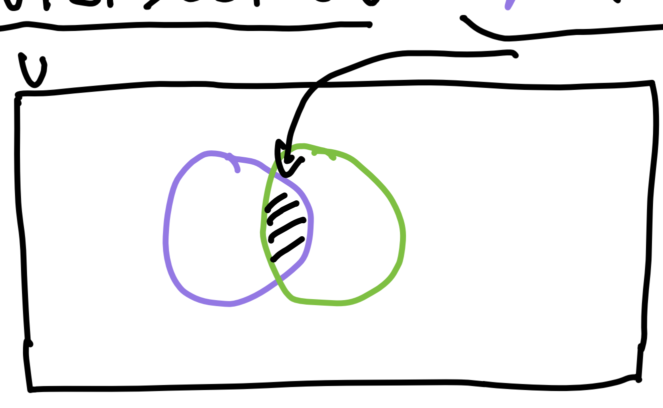
SET OPERATIONS

* **UNION:** $A \cup B$: the set formed from all the elements of A and all the elements of B



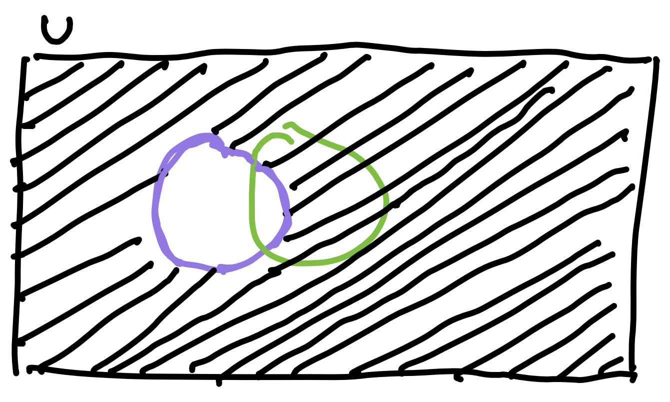
overlap = repetitions that appear in both sets are counted only once

* **INTERSECTION:** $A \cap B$: the set of elements both in A and in B



(Slido asker point out can also be written A')

* **COMPLEMENT:** $\overline{A}$: all the elements (of the universe U) that are NOT in A



All of these defs can be written using set builder notation:

("|" reads "such that")

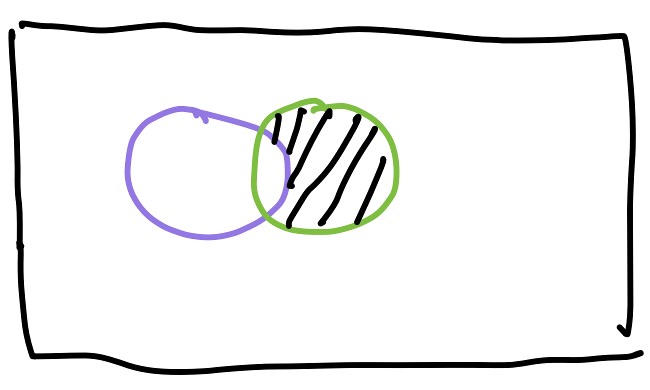
$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\} = \{x \mid x \in A \text{ or } x \in B\}$$

(alternate acceptable writing)

$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$$

$$\overline{A} = \{x \in U \mid x \notin A\}$$

* **DIFFERENCE:** $B \setminus A$: elements of B that are not in A



$$B \setminus A = \{x \in U \mid x \in B \text{ and } x \notin A\}$$

⚠ Fun fact for Slido asker: $B \setminus A = B \cap \overline{A}$ (difference)
(something that can be proven)