Lecture 1 : Sets

Sets are:
- a collection of "things" called elements (or items)
- order does NOT matter
  $\{1,2,3\} \equiv \{2,1,3\}$
- repetitions do NOT matter
  $\{1,1,1,2,4\} \equiv \{1,2,2,4\}$
  ↳ symbol for "equivalence" (the objects are the same)

Notations

- Sets use braces $\{\ \}$
- If S is a set and x is an element then we write $x \in S$ which is read as
  - "x belongs to S"
  - "x is included in S"
- SET ROSTER: all elements are specified (potentially with ellipses ... when sequences are obvious)

$\{1,2,3\}$ $\{A, 5, \text{"hello"}\}$ $\{10, 11, 12, 13, 14, 15, 17\}$

$O_{13} = \{1, 3, 5, \dots, 13\} \equiv \{1, 3, 5, 7, 9, 11, 13\}$

$N_{10} = \{1, \dots, 10\} \equiv \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

↳ ellipses indicate that we include the full sequence 2, 3, 4, 5, 6, 7, 8,

equivalent

- SET BUILDER: specify a set by starting with some initial set and applying conditions

$O_{13} = \{ x \in \mathbb{N},\ 1 \leq x \leq 13,\ x \text{ is odd} \}$

initial set | condition 1 | condition 2

order does not matter : AND

- SPECIAL SETS:
  - $\mathbb{N}$ natural numbers $0, 1, 2, 3, 4, 5 \dots$ positive infinity
    - ⚠ Some sources EXCLUDE zero $\emptyset$ but in this course $\emptyset \in \mathbb{N}$
    - ⚠ $\mathbb{D} \subset \mathbb{N}$ $\emptyset \notin \mathbb{N}^*$ ← $\mathbb{N}^*$ is $\mathbb{N}$ with the zero removed
  - $\mathbb{Z}$ integers $\dots -3, -2, -1, 0, 1, 2, 3 \dots$
  - $\mathbb{Q}$ rational numbers (FRactions) (quotients) $\frac{p}{q}$ with $p \in \mathbb{Z}$, $q \in \mathbb{N}^*$; $\frac{1}{1} = \frac{-1}{-1}$
    ↳ the denominator should
    - not be zero
    - not change the sign
  - $\mathbb{R}$ real numbers $0, 1.7, \frac{5}{3}, \sqrt{2}, \pi = 3.14568\dots$
- EMPTY SET: $\emptyset$ this set contains no elements
  $\{\}$ equivalent notation

⚠ Very frequent confusion with three diff. concepts:

0	the number zero	
$\{0\}$	the set containing the number zero	
$\emptyset$	the empty set	$\emptyset \equiv \{\}$

⚠ Confusion

$\mathbb{D}$	the number zero	(oval)
$\emptyset$	the empty set	(circle)

- Connected topics
  - a set that allows repetitions is a multiset
  - a set where order matters is a sequence (as well as repetitions)

SUBSETS

When we write

$$A \subset B$$

we mean "A is a subset of B" which means that every element of A is also an element of B

↳ if $x \in A$ then $x \in B$ (important to know later)

⚠ Two distinct notations

			(analogous to:
$\subset$	strict subset	$\{1\} \not\subset \{1\}$	$<$
$\subseteq$	subset or equal	$\{1\} \subseteq \{1\}$	$\leq$

$\{1,2\} \subset \{1,2,3,4,5\}$
$\{1,2\} \subset \mathbb{N}$
$\{1,2\} \subset \mathbb{Z}$

$\emptyset \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
↳ the special sets are subsets of each other

$\emptyset$ the empty set is a subset of every set

$\{1,2\} \subset \{1,2,\emptyset\}$ but $\{1,2,\emptyset\} \not\subset \{1,2\}$
$\{1,2\} \subset \{1,2,\emptyset,\mathbb{N}\}$

Being a subset ≠ being an element

$\{\} \subset \{1,2\}$ $\{\} \notin \{1,2\}$

the empty set being a subset says nothing about whether it is an element

$\{\} \in \{1, 2, \{\}\}$

$A = \{1,2\}$
$B = \{1,2,\{1,2\}\}$

$A \subset B$ $A \not\subset B$
$A \in B$ $A \notin B$
$\emptyset \subset B$ $\emptyset \notin B$

$\{1,2\} \subseteq \{1,1,2,2\}$
$\{1,1,2,2\} \subseteq \{1,2\}$
$\{1,2\} \equiv \{1,1,2,2\}$

$\{1,2\} \not\subset \{1,1,2,2\}$
(because they are equivalent as repetitions don't matter)