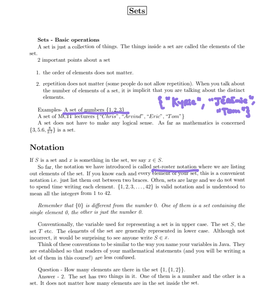
Lecture 1: Sets



Sets

Sets - Basic operations

A set is just a collection of things. The things inside a set are called the elements of the set.

2 important points about a set.

1. the order of elements does not matter.
2. repetition does not matter (some people do not allow repetition). When you talk about the number of elements of a set, it is implicit that you are talking about the distinct elements.

{"Kyoto", "Tallinn", "Tunis"}

Examples: A set of numbers $\{1, 2, 3\}$.
A set of CITIES $\{$"Kyoto", "Tallinn", "Tunis"$\}$.
A set does not have to make any logical sense. As far as mathematics is concerned $\{3, 5.6, \frac{\text{Pi}}{2}\}$ is a set.

Notation

If S is a set and x is something in the set, we say $x \in S$.
So far, the notation we have introduced is called set-roster notation where we are listing out elements of the set. If you know each and every element in your set, this is a convenient notation i.e. just list them out between two braces. Often, sets are large and we do not want to spend time writing each element. $\{1, 2, 3, \ldots, 42\}$ is valid notation and is understood to mean all the integers from 1 to 42.

Remember that $\{0\}$ is different from the number 0. One of them is a set containing the single element 0, the other is just the number 0.

Conventionally, the variable used for representing a set is in upper case. The set S, the set T etc. The elements of the set are generally represented in lower case. Although not incorrect, it would be surprising to see anyone write $S \in x$.
Think of these conventions to be similar to the way you name your variables in Java. They are established so that readers of your mathematical statements (and you will be writing a lot of them in this course!) are less confused.

Question - How many elements are there in the set $\{1, \{1, 2\}\}$.
Answer - 2. The set has two things in it. One of them is a number and the other is a set. It does not matter how many elements are in the set inside the set.

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Sets are:

- a collection of things called elements
- order of elements does not matter

$\{1, 2, 3\} \equiv \{2, 3, 1\}$ ← means "is equivalent"

- repetition does not matter (it is ignored usually)

$\{1, 1, 1, 2, 3\} \equiv \{1, 2, 3\}$

(when discussing the number of elements of a set, it refers to the distinct elements)

Notations

- Sets use braces { }
- If S is a set and x is an element Then $x \in S$ means that x belongs to S or x is included in S.
- SET ROSTER: all elements are specified, potentially with ellipses when sequences are obvious

$\{1, 2, 3\}$ $\{A, 5, \text{"hello"}\}$ $\{1, 2, 3, 4, 5, 6\}$

$\{1, \ldots, 6\}$ ← ellipses indicate we include the full sequence 1, 2, 3, 4, 5, 6

$\{1, 3, \ldots, 11\}$ ← the pattern is 1, 3, 5, 7, 9, 11

- SET BUILDER notation: specify a set by starting with a universal set and applying conditions.

$\{x \in \mathbb{N}, 1 \leq x \leq 11\}$ ← this comma reads as "such that" ← lesser or equal to

can also be a vertical line: $\{x \in \mathbb{N} \mid 1 \leq x \leq 11\}$ equivalent notation

- SPECIAL SETS:
  - $\mathbb{N}$ natural numbers $0, 1, 2, 3, 4, 5, \ldots$

⚠ Some people say that $0 \in \mathbb{N}$ and others disagree — source of great confusion

⚠ $0 \in \mathbb{N}$ $0 \notin \mathbb{N}^*$ ← use the star to remove 0

  - $\mathbb{Z}$ integers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$
  - $\mathbb{Q}$ Rational numbers $\frac{p}{q}$ $p \in \mathbb{Z}$, $q \in \mathbb{Z}$, $q \neq 0$
  - $\mathbb{R}$ Real numbers 0, 1.7, $\frac{1}{3}$, π, $\sqrt{2}$

Bonus notation

$\mathbb{Z}^+ = \{x \in \mathbb{Z}, x > 0\}$

$\mathbb{Z}^- = \{x \in \mathbb{Z}, x < 0\}$ ← using the equal sign because this is a definition

- EMPTY SET: $\emptyset$ this set contains no elements, it can also be noted $\{\}$

⚠ Frequent confusion: Three different concepts

0 (the number) $\neq$ $\{0\}$ (the set containing the number)

$\{0\}$ $\neq$ $\emptyset$ (the empty set that doesn't contain any elements)

Question: Is 1.0 a rational number? YES.

$\mathbb{N}$ is a subset of $\mathbb{Z}$ is a subset of $\mathbb{Q}$ ... every element of $\mathbb{N}$ is also an element of $\mathbb{Z}$

- SUBSETS: when we write

$A \subset B$

we mean "A is a subset of B", which means that every element of A is also an element of B

↳ IF $x \in A$ THEN $x \in B$ (important later)

⚠ $A \subseteq B$ does not imply $B \subset A$

$\{1, 2\} \subseteq \mathbb{Z}$ (A, B)

$1 \in A \rightarrow 1 \in B$ ✓
$2 \in A \rightarrow 2 \in B$ ✓
but $3 \in \mathbb{Z}$ but $3 \notin \{1, 2\}$

$\subset$ strict subset $\{1\} \not\subset \{1\}$

$\subseteq$ subset or equal $\{1\} \subseteq \{1\}$

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