Penn

# CIT 5920–Mathematical Foundations of Computer Science
# Homework 2: *Introduction to counting*

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Complete the online portion on PrairieLearn. For the written section, use the HW1 template on Overleaf, shared on the course forum and Canvas. Each exercise should be on a separate page. Only submissions in this format will be accepted. Submit your written work on Gradescope by the deadline. For assistance or inquiries, don't hesitate to: Attend office hours; post questions on the class forum; ask about the motivation behind this material.

Guidelines:

- Ensure clarity in your answers.
- Avoid direct answers unless specified. Merely providing a number will result in deductions.
- Clearly state any assumptions in combinatorics questions. Reasonable assumptions will be credited. For instance, assuming two people are identical is not reasonable, neither is assuming that the order matters when mixing paints.
- Prefer math symbols over words, e.g., use $A \cap B$ instead of "The elements common to both A and B."
- If unsure about the length or style of your answer, consult during office hours or post on the class forum. However, we won't provide direct answers.
- For the PrairieLearn section, consider using Wolfram Alpha's Equation Solver. For the written section, avoid calculators. Answers can be in factorial or $\binom{n}{k}$ form.

## Exercise 0 – PrairieLearn Questions [ 9pts ]

Use the QR code to the right to access the PrairieLearn portion of this homework. **Please login using your Penn Google account.**

1. Most questions are designed to provide you with an infinite number of variations.
2. With these questions, if you respond incorrectly, you will have the opportunity to try again until you get the question right. To earn some credit on the question, you must answer *any* variant from the first try. To earn full credit, you must answer multiple variants to earn full credit (and to encourage you to be careful, you can earn points faster when you have a consecutive streak of correct answers).

## Exercise 1 – Checking Relation Properties [ 0.5 x 5 x 4 = 10pts ]

For each of the following relations, tell us whether they are reflexive, symmetric, transitive and anti-symmetric.

- When the answer to a property is YES: Provide very brief explanations.
- When the answer to a property is NO: Provide a single counter-example.

For instance, on the set $\{a, b\}$, the relation $X = \{(a, a), (a, b)\}$ is:

- **reflexive?** NO, the relation $X$ is not reflexive since we can find an element from $\{a, b\}$ that is not in relation with itself: $(b, b) \notin X$;

- **symmetric?** NO, the relation $X$ is not symmetric since $(a, b) \in X$ but not the symmetric pair $(b, a) \notin X$;
- **transitive?** YES, $aXa$ and $aXb$ implies $aXb$ (the only thing is since there are only two elements, this example is not compelling).

A. On the set $X_1 = \{1, 2, 3\}$ the relation $R_1 = \{(1, 3), (3, 1)\}$

B. On the set of all humans, two humans being related if they are friends on Facebook.

C. On the set of all humans, two humans $h_1$ and $h_2$ being related by $R_3$ in the following way: $h_1 R_3 h_2$ if and only if $h_1$ follows $h_2$ on Twitter.

D. On the set of all files on your hard disk, two files being related if and only if they are in the same folder (directory).

E. On the set of $X_5 = \{11, 2, 3, 4, 5\}$, the relation $R_5 = \{(11, 11), (11, 2), (2, 11), (3, 4), (4, 3)\}$

### Exercise 2 – Generating Relations [ 4pts ]

List out all possible *reflexive* relations on the set $A = \{a, b\}$. No explanation is needed, though note that when we are asking for relations, we do not need a mathematical formula. Indeed recall that a relation is just a collection of pairs.

### Exercise 3 – Symmetry and Anti-symmetry [ 2pts ]

A student is asked to show a relation (assume the relation is not empty) is anti-symmetric and they use the following argument:

- they show the relation to be symmetric;
- they then say: "*The relation is symmetric, therefore it cannot be anti-symmetric.*"

Is the student's logic correct? If there is a flaw in the logic, please explain what the flaw is, and provide a counter-example to illustrate your argument.

### Exercise 4 – Combinations of Apartments [ 2pts ]

Kunal and Daria want to live in studio apartments (no roommates allowed). They decide to rent in the same building. The building has apartments numbered 301, 302, ... 309. How many distinct possibilities exist for the apartments they end up renting? *Hint: You obviously care about who got which apartment.*

### Exercise 5 – Counting and Problem Modeling with Smoothies [ 3pts ]

The smoothie cart right next to the Quad entrance on Spruce Street allows you to pick and choose any fruits and then they put it in a blender and make a smoothie.

Assume they have the following fruit: Oranges, strawberries, blueberries, raspberries, peaches, mangoes, bananas, pineapples and kiwis.

Assume also that you do not get to order a "2 kiwis, 1 mango, 4 blueberries" smoothie. Rather, you order a "kiwi, mango and blueberry" smoothie. In other words, each fruit is either included in the smoothie or not, but we do not worry about the quantity.

Assume you want to live life on the wild side and try every possible smoothie. You do not want to waste your time trying a smoothie with just one fruit (technically, is that even a smoothie??), or a smoothie with no fruit. How many days will it take for you to accomplish your mission if you have a smoothie for breakfast and lunch (not sure they are around for dinner)!

### Exercise 6 – Counting Constrained Functions [ 3pts ]

Consider all functions from the set $X = \{a, b, c, d, e\}$ to the set $Y = \{p, q\}$.

A. How many onto functions are there from $X$ to $Y$? Do this without having to explicitly list out all functions.

B. How many 1-to-1 functions from $X$ to $Y$ are there?

### Exercise 7 – Counting and Problem Modeling with Cookies [ 2pts ]

6 MCIT students go to Insomnia Cookies. Insomnia has 10 cookies each of Chocolate Chip, Peanut Butter, White Chocolate Macadamia, and Oatmeal Raisin. Each of them orders a single cookie. The next day they were asked by Arvind "Who ate what?". How many distinct possible answers are there?

### Exercise 8 – Counting and Problem Modeling with Courses [ 3pts ]

As part of the MCIT curriculum you have to do 6 core courses and 4 electives. Your goal in this question is to figure out the number of ways to complete the degree.

For the core courses

- You can take 591 or 590.
- 592.
- 593 or 501.
- 594
- 595 or 505.
- You can take 596 or 502

*There are actually more nuances to these rules. Let us not worry about them while doing the question.*

For the electives, you have a giant list to choose from but really there seem to be a few that are popular, so for the purposes of this question we will say that for electives here are the possible choices - Databases (550), Big data (545), Machine Learning Intro (519), Machine Learning Advanced (520), and Internet and Web Systems (555).

In how many ways can you complete your degree? To be clear, we are NOT distinguishing between the order in which you do the courses, the number of years to take to finish the course etc. We just care about what course numbers appear on your transcript.

### Exercise 9 – Constrained Enumeration by Complement [ 4pts ]

Passwords consist of character strings of 6 to 8 characters. Each character is an English letter or a digit from 0 through 9. Each password must contain at least one digit and at least one upper case character. How many passwords are possible?

*Hint: It is useful to use the idea of the complement of a set here*

### Exercise 10 – Constrained Sequences [ 2pts ]

Ten members of a club are lining up for a photo. The club has specific roles: one president, one VP, one secretary, and one treasurer. Determine the number of ways they can line up based on the given constraints.

A. How many ways are there to line up the ten people without any constraints?

B. How many ways can they line up if the VP must stand beside the president?

C. How many ways can they line up if the president must stand beside the secretary and the VP must stand beside the treasurer?

 