CIT 5920–Mathematical Foundations of Computer Science
Homework 1: *Sets, set operations and relations*

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Please complete the PrairieLearn portion online, and use the HW1 template on Overleaf that we have also shared on the course forum and Canvas, with one exercise per page, for the written portion of this homework. We will not accept any other format of written submission. You can submit your written portion on Gradescope by the due date.

Please don't hesitate to come to office hours, ask questions on the class forum, and don't hesitate to ask use about the motivation of this work.

Exercise 0 – PrairieLearn Questions [8pts]

Use the QR code to the right to access the PrairieLearn portion of this homework. **Please login using you Penn Google account.**

1. Most questions are designed to provide you with an infinite number of variations.
2. With these questions, if you respond incorrectly, you will have the opportunity to try again until you get the question right. To earn credit on the question, you must answer *any* variant from the first try.

Exercise 1 – Real Life Sets [3pts]

Consider the following sets that have already been defined:

- F, the set of all CIS faculty at Penn. The list is here - click here.
- R, the set of libraries at Penn. The list is here - click here.

Using those definitions, answer the following questions.

A. What is the value of $|R|$, the cardinality of the set of libraries at Penn? (You may ignore *Associated Libraries*, and treat each row as an element of the set.)

B. Write the following set in roster form (consider all faculty listed on the page to be faculty, and to find out what courses faculty are teaching, you may visit `courses.upenn.edu`):

$$T(F, \text{CIS/CIT } 5000) = \{f \in F \mid f \text{ is teaching a CIS/CIT 5000 level course in Fall 2024}\}$$

This question may be more or less time consuming than appears. This is part of the question.

C. Write the following set in roster form:

$$\{p \in \mathbb{Z} \mid p^2 = p\}$$

Exercise 2 – Set Proof [2pts]

Using the properties of sets mentioned in class (and available at the end of this handout), show why the following is true.

$$A \cup (\emptyset \cap B) = A$$

Exercise 3 – Set Index Notation and Formal Union [4pts]

Let $\mathbb{Z}_i$ be used to represent integers that are divisible by i. Remember that we say a is divisible by b, if a divided by b will leave a remainder of 0. For instance,

$$\mathbb{Z}_4 = \{\ldots, -16, -12, -8, -4, 0, 4, 8, 12, 16, \ldots\}$$

Given that notation, compute what the following expressions evaluate to.

*This question is designed mainly to get you to slowly overcome fears of mathematical notation. Please express the answer as mathematically as possible. Please provide a **brief explanation**.*

A. $\mathbb{Z}_2 \cap \mathbb{Z}_3$

B. $\bigcup_{i=1}^{10} \mathbb{Z}_i$

C. What is a simple concise name for the set $\mathbb{Z} \setminus \mathbb{Z}_2$?

Exercise 4 – Set Builder Notation [4pts]

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

A. $\{-2, -1, 0, 1, 2\}$

B. $\{3, 6, 9, 12, ...\}$

C. $\{-3, -1, 1, 3, 5, 7, 9\}$

D. $\{0, 10, 20, 30, ..., 1000\}$

Exercise 5 – Set Identity Proofs [3pts]

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

The purpose of this is to help you start developing your ability to write proofs. Here, the set identities are "Lego Blocks". It is not so important to learn them by heart, but rather, to develop a knack to understand how to stack them together.

For example, let's do the first statement:

$$(\overline{A} \cap C) \cup (A \cap C) = C$$

You would stack the Distributive Law, the Complement Law of Union, and Identity Law of Intersection:

$$\begin{aligned}
(\overline{A} \cap C) \cup (A \cap C) &= (\overline{A} \cup A) \cap C && \text{[Distributive Law]} \\
&= U \cap C && \text{[Complement Law of Union]} \\
&= C && \text{[Identity Law of Intersection]}
\end{aligned}$$

A. $(\overline{A} \cap C) \cup (A \cap C) = C$

B. $(B \cup A) \cap (\overline{B} \cup A) = A$

C. $\overline{A \cap \overline{B}} = \overline{A} \cup B$

D. $\overline{A} \cup (A \cap B) = \overline{A} \cup B$

E. $A \cap (B \cap \overline{B}) = \emptyset$

F. $A \cup (B \cup \overline{B}) = U$

Set Identities

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination laws	$A \cap \emptyset = \emptyset$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \emptyset$ $\overline{U} = \emptyset$	$A \cup \overline{A} = U$ $\overline{\emptyset} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$